

Contributions to change-point analysis under long-range dependencies

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Diplom - Ökonom Juliane Willert
geboren am 17. Februar 1984 in Bergen auf Rügen

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Kurzzusammenfassung

Die Analyse von Strukturbrüchen umfasst die Ermittlung von Anzahl und Position von strukturellen Brüchen in Zeitreihen. Durch das wachsende Interesse im Finanzmarktbereich in den letzten 50 Jahren wurde die Forschung für das korrekte Auffinden von Brüchen immer wichtiger, um entsprechende Prozesse präzise modellieren, testen und prognostizieren zu können. Diese 4 Beiträge untersuchen verschiedene Ansätze bei dem Vorherrschen von langfristigen Abhängigkeiten in den Zeitreihen, auch bezeichnet als langes Gedächtnis.

Kapitel 2 und 3 basieren auf dem Ansatz der atheoretischen Regressionsbäume (ART). In einem ersten Schritt wird ein stark angepasster Baum aufgespannt, der die potentiellen Bruchpunkte enthält. Er entsteht durch das Anpassen von stückweise linearen Funktionen an die Zeitreihe. In dem zweiten Schritt wird die Überanpassung korrigiert mit Hilfe einer Zurückschneideprozedur, die die Äste mit dem geringsten Erklärungsbeitrag entfernt. In Kapitel 2 wird gezeigt, dass das häufig verwendete BIC (Bayesianische Informationskriterium) als Zurückschneideprozedur unter langem Gedächtnis nicht gut arbeitet aufgrund seines zu schwachen Strafterms. Eine einfache, aber effektive Methode für das Zurückschneiden wird vorgestellt, die den zu geringen Einfluss des Strafterms entsprechend ausgleicht. In Kapitel 3 (gemeinsam verfasst mit Philipp Sibbertsen) wird eine Modifikation des BIC, das LWZ (Liu, Wu und Zidek (1997)), vorgestellt, welches die gut erforschten Eigenschaften des BIC und die Besonderheiten bei langem Gedächtnis miteinander vereint. Dies wird nun mit alternativen Zurückschneideverfahren wie dem BIC und dem LIC (Lavielle und Moulines (2002)) verglichen und Konsistenz der Schätzung auf Basis der atheoretischen Regressionsbäume kann gezeigt werden. ART stellt sich als überaus schneller Ansatz zur Schätzung der Anzahl und Position von Strukturbrüchen heraus.

Die folgenden Beiträge in Kapitel 4 und 5 befassen sich mit Problemen der Strukturbruchanalyse in Bezug auf die Testverfahren CUSUM und MOSUM. Zusätzlich zum Mittelwert einer Zeitreihe kann ebenfalls der Lange-Gedächtnis-Parameter zeitabhängig sein. In Kapitel 4 (gemeinsam verfasst mit Philipp Sibbertsen) wird ein CUSUM-Quadrat-Test basierend auf Leybourne et al. (2007) verwendet, um das Verhalten des langen Gedächtnis gegen einen Bruch in diesem zu testen. Die Testalternative umfasst den Bruch in der Persistenz sowohl vom stationären in den instationären Bereich als auch umgekehrt. Bedauerlicherweise ist diese Testprozedur nicht robust gegenüber zusätzlichen Brüchen im Mittelwert und erleidet starke Verzerrungen in der Size. Deshalb sind adjustierte kritische Werte unerlässlich, wenn bekannt ist, dass ein Mittelwertbruch im datengenerierenden Prozess vorliegt.

Ein anderer Ansatz bezüglich des Zustand abhängigen Verhaltens von Parametern wird im abschließenden Kapitel 5 (gemeinsam verfasst mit Florian Heinen) beleuchtet. Die Testidee des CUSUM Testes wird modifiziert zu einem Monitoring-Ansatz. Dieser erlaubt die schnelle Entdeckung einer Änderung in der langfristigen Abhängigkeitsstruktur, also dem Parameter des langen Gedächtnis. Der MOSUM Test kann unproblematisch erneut ausgeführt werden, so bald neue Daten vorliegen, ohne in Probleme des multiplen Testens zu geraten.

Schlagwörter: Langes Gedächtnis, Strukturbrüche, ART, Informationskriterien, CUSUM, MOSUM

Short summary

In time series analysis the change-point analysis describes the detection and localization of structural breaks. During the last 50 years the growing interest in financial markets nourished the research in finding breaks in different parameters to model, test and forecast the underlying process correctly. These four contributions investigate different approaches when it comes to long-range dependencies, named long-memory behavior.

Chapter 2 and 3 focus on approaches based on atheoretical regression trees (ART). In the first step a tree is constructed well overfitted with potential breakpoints due to the fitting of piecewise constant functions to the time series. In the second step the overestimation is adjusted through a pruning procedure that cuts back branches with the lowest contribution. In chapter 2 it is shown that the bayesian information criterion (BIC), which is commonly used as a pruning method, does not operate well in the long memory framework because of an inferior penalty term. A simple but effective procedure is presented to deal with this underweight impact of the penalty term. In chapter 3, co-authored with Philipp Sibbertsen, a modification of the BIC, the LWZ (Liu, Wu and Zidek (1997)), is presented to overcome long-range dependence issues and use the well-researched properties of the BIC at the same time. It is compared to alternative pruning criteria like the BIC or LIC (Lavielle and Moulines (2002)). Also consistency of the estimation using tree-based methods is shown. ART are highlighted as a fast approach for change-point detection that can estimate the number and location of structural breaks both in a single algorithm with minor impacts through long memory behavior.

The following essays in chapter 4 and 5 overcome problems regarding change-point analysis in the context of CUSUM and MOSUM testing. Based on the idea that not only the mean is at risk of changing over time the long memory parameter could additionally be time-dependent. In chapter 4, co-authored with Philipp Sibbertsen, the CUSUM-squared based test for a change in persistence by Leybourne et al. (2007) tests long memory behavior versus a break in persistence from stationary to non-stationary long memory and vice versa in the alternative. Unfortunately this test procedure is not robust against shifts in the mean and suffers from serious size distortions when mean shifts occur. Therefore, adjusted critical values are needed when it is known that the data generating process has a mean shift.

A different perspective on the regime changing behavior is taken in the concluding chapter 5, co-authored with Florian Heinen. The CUSUM idea is modified to a monitoring technique that allows the detection of a single change in the long-run correlation structure of a time series at some unknown future point in time. The MOSUM test can be executed once new data arrives without running into multiple testing problems. Different forms of boundary functions for the test are derived and the finite sample performance is investigated.

Keywords: long memory, structural breaks, ART, information criteria, CUSUM, MOSUM

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Chapter 1

Introduction

Introduction

In the last 50 years the study of detection and location of structural breaks in time series developed effectively both in the statistical and econometric literature. The growing interest in financial markets and at the same time strong shocks like world wars and global economic and oil crises led to the awareness that conventional time series models were not sufficient any longer. Change-point analysis became an area of research that gained attention and spread out not only in finance but in medicine, chemistry, meteorology, physics, computer science and engineering.

Early contributions to the change-point analysis made by Chow (1960), who suggested a test for structural break detection at a known date, and Brown et al. (1975), who developed the theory for tests of significance for cumulative summation (CUSUM), laid the foundation for multiple break detection. By examining macroeconomic time series Nelson and Plosser (1982) began to model the mean through a stochastic model rather than a deterministic trend and later Perron (1989) modeled breaks for explaining shocks. He found out that a misspecification of these shocks would bias unit root tests. Zivot and Andrews (1992) reconsidered Perron's findings and saw disadvantages in his choice to set the breakpoints. Therefore Zivot and Andrews (1992) modeled their own breakpoint estimator. On that basis Andrews (1993) proposed a test for detection of a break at unknown break dates however with average sample properties. For further overviews see Hansen (2001) and Banerjee and Urga (2005).

More recently Bai and Perron (1998, 2003) extended their work to a multiple breakpoint estimator when the date is unknown. Their estimator, later referred as the Bai-Perron-estimator, provides a basis for change-point analyses not only for time series. It is a consistent estimator with good small sample properties and can serve as a benchmark when it comes to breakpoint detection. However, the Bai-Perron-estimator is computationally intensive and therefore not feasible for long time series (Cappelli et al. (2008) see more than 600 time points too long as a rule of thumb). Additionally the Bai-Perron-estimator depends on the pre-specification of the maximum number of breaks which increases the computational time disproportionately with larger maxima. Moreover recent studies have shown that the Bai-Perron-estimator is unsuitable for long memory time series (Rea (2008)).

Long-range dependency models have been most successful for economic time series. Large evidence for the effective modeling based on long memory processes can be found at e.g. Christensen and Nielsen (2007), Shimotsu (2006), Bhardwaj and Swanson (2006), Deo et al. (2006), Hurvich et al. (2005), Granger and Hyung (2004), Breidt et al. (1998) and Andersen and Bollerslev (1997). The persistence indicates local trends and long cycles and can handle e.g. long-term dependencies on financial markets. On the other hand this behavior makes it very challenging for breakpoint detection procedures to find the correct breaks (see Sibbertsen (2004)). The biggest challenge is to distinguish between true long memory behavior and regular breaks in the mean and inevitably the literature started discovering spurious long memory behavior. For

instance Perron and Qu (2010), Granger and Hyung (2004), Gouriéroux and Jasiak (2001) and Diebold and Inoue (2001) find examples where long memory can easily be confused with breaks in the mean and conclude that the distinguishing is very hard because both processes are almost observationally equivalent (Shimotsu (2006)). Nevertheless they do not consider that certain behavior can be explained through different modeling approaches and it does not indicate the true data generating process. Choi and Zivot (2007) showed that even after adjusting for breaks in the mean there is still substantial evidence for long memory. That's why it is so crucial to detect all mean shifts regardless of the persistence in order to avoid misleading conclusions. The estimation of the long memory parameter e.g. is heavily biased when there are structural changes in the mean or in the long memory parameter itself (see Granger and Hyung (1999) and Diebold and Inoue (2001)). What makes it even more appealing for current research is the fact that the well-established Bai-Perron-estimator tends to fail finding the correct number and location of breakpoints when it comes to high persistence and hence reasonable alternatives are required (Rea (2008)).

This thesis focusses on detecting structural breaks in the mean occurring at unknown dates when there is long-term persistence, named long-memory behavior. To this purpose the use of a fast non-parametric procedure based on regression trees is suggested. In the first step the tree is spanned and constructs a well overfitted tree with potential breakpoints due to the fitting of piecewise constant functions to the time series. In the second step the overestimation, especially for short series, is adjusted through a pruning procedure that cuts back branches with the lowest contribution to the deviance reduction to gain the optimal partitioning. This binary splitting in time series analysis was first justified by Hartigan (1975) and later Breiman et al. (1993) derived the large sample theory that is seen by Wu et al. (2008) as a preferred method when it comes to partitioning. When applying atheoretical regression trees (ART) to time series some open questions following Rea et al. (2010) are:

- What is the best tree selection and pruning procedure?
- Do ART find or add breaks through the fitting of piecewise constant functions?
- What are the effects of serial correlation on the performance?
- Are ART robust to any kind of noise structure or a lack of breaks?
- How do ART handle long-range dependencies?

The first two questions are fundamental for the breakpoint estimation. How to construct optimal break detection procedures and whether it provides consistent estimates for the number and location of the breaks is the key element of change-point analysis. The robustness can be checked via monte carlo studies. Obviously an increase in the length of the series leads to more robust results. The natural focus here marks the impact on the performance of ART when it comes to long memory behavior.

In chapter 2 and 3 two different approaches for the pruning procedure will be introduced. Both show good robustness properties when it comes to serial correlation and long-range dependencies and reveal superior performance to alternatives. In chapter 2 it is shown that the bayesian information criterion (BIC), which is commonly used as a pruning method, does not operate well in the long memory framework because of an inferior penalty term. A simple but effective procedure is presented to deal with this underweight impact of the penalty term. In chapter 3, co-authored with Philipp Sibbertsen, a modification of the BIC, the LWZ (Liu, Wu and Zidek (1997)), is presented to overcome long-range dependence issues and use the well-researched properties of the BIC at the same time. It is compared to alternative pruning criteria like the BIC or LIC (Lavielle and Moulines (2002)). Also consistency of the estimation using tree-based methods is shown. ART are highlighted as a fast approach for change-point detection that can estimate the number and location of structural breaks both in a single algorithm with minor impacts through long memory behavior.

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A different perspective on the regime changing behavior is taken in the concluding chapter 5, co-authored with Florian Heinen. The CUSUM idea is modified to a monitoring technique that allows the detection of a single change in the long-run correlation structure of a time series at some unknown future point in time. The MOSUM test can be executed once new data arrives without running into multiple testing problems. We focus on the detection of an increasing persistence with a process that is becoming non-stationary under the alternative. Different forms of boundary functions for the test are derived and the finite sample performance is investigated. The concluding application shows that loss of controllability indicated through an increasing persistence is indeed a highly probable outcome for economic time series.