# Status Concerns, Present-Bias and the Public Provision of Private Goods

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**Abstract** To a substantial degree, governments provide private goods to their citizens – like education, health care, or child care. In this thesis, the theory of publicly provided private goods is extended by insights from sociology, social psychology, and the recent literature on behavioral economics. The implications of two types of non-standard preferences are studied: concerns for social status and present-bias. The thesis includes both normative and politico-economic contributions.

**Keywords:** Public Provision of Private Goods, Status Concerns, Present-Bias.

**Kurzzusammenfassung** Staaten stellen in erheblichem Umfang private Güter bereit – wie etwa Bildung, Gesundheitsleistungen oder Kinderbetreuung. In dieser Dissertation wird die Theorie der staatlichen Bereitstellung privater Güter um Ansätze aus der Soziologie und Verhaltensökonomik erweitert. Insbesondere werden die Auswirkungen von Statusmotiven und einer übermäßigen Gegenwartspräferenz (sog. "Present-Bias") untersucht. Die Arbeit liefert sowohl politökonomische als auch normative Beiträge.

**Schlagwörter:** Staatliche Bereitstellung privater Güter, Statuspräferenzen, Present-Bias.

# Contents

# Chapter 1

## Introduction

### 1.1 Motivation

In most countries, a significant share of the public budget is devoted to the provision of *private* goods, i.e., goods that are excludable and rival in consumption. Prominent examples include education, health care, or old-age consumption, but many governments also provide child care, housing, nutritional assistance, transportation, or care of the elderly. The volume of public spending on private goods ranges from 5 percent of GDP in developing countries up to 25 percent in some members of the OECD (Bearse et al., 2000; Currie and Gahvari, 2008). Typically, government-provided goods are made available free of charge and at a uniform level. Most of them are (or in principle can be), however, also traded on markets. For example, people can send their children to private schools or kindergartens, buy private health insurance and consult private physicians. A fundamental question is then whether there are any rationales for the *public* provision of specific private goods.

From a traditional economic perspective, the answer is negative (see, e.g., Aaron and von Fürstenberg, 1971). The standard argument against public provision is based on the observation that it is an in-kind transfer and may constrain consumption choices: people might choose different levels of a publicly provided good if they receive the value of the in-kind transfer in cash. Replacing public provision by equal-valued cash payments should thus (weakly) increase welfare – as people could then select the consumption bundles that best fit their preferences.

In the past decades, a number of possible rationales for public provision have been put forward, including both normative and politico-economic arguments (see Currie and Gahvari, 2008, for a survey). Most existing explanations follow the convention of traditional economic theory and assume "standard" preferences: people are only interested in their own consumption of goods and services and are fully rational and make no systematic errors. However, insights from sociology, social psychology, and the recent literature on behavioral economics suggest that actual preferences deviate from what is assumed in standard economic models. There is now a substantial body of empirical evidence in line with the idea that people have status concerns. I.e., beyond absolute levels of consumption, individuals care about relative positions (of income, wealth, or the consumption of particular goods) and about how they are perceived and evaluated by others (Truyts, 2010; Frank and Heffetz, 2011). In addition, evidence from laboratory and field experiments documents various biases in individual decision making (DellaVigna, 2009). In particular, when faced with intertemporal decisions, many people are biased towards the present and assign higher relative weight to well-being in the current period than they did if asked in any period before (Fredrick et al., 2002).

The aim of this thesis is to study how status concerns and present-biased preferences affect the theory of publicly provided private goods. As shown by a growing theoretical literature, allowing for non-standard preferences can considerably alter the policy conclusions of standard economic models (see, for example, Bernheim and Rangel, 2007). The vast majority of previous work focuses on issues of taxation or the provision of public goods. However, status seeking and present-bias may also have important implications for private good provision.

First, many government-provided goods can alternatively be purchased on markets. Status concerns – i.e., feelings of distinction, prestige, or even stigma – affect the choice between a publicly provided good and its private alternatives. For instance, taking one's children out of public schools and sending them to private institutions may satisfy needs of elitism and exclusiveness or conspicuously signal higher status. Conversely, take-up of social housing, nutritional assistance, and public transportation is often stigmatized. As levels for publicly provided goods are determined politically, status concerns may have repercussions on the political economy of public provision.

Second, individuals who strive for status will spend resources to achieve (or maintain) it. This spending is inherently positional as each individual's status depends on how much others consume of goods that confer a high relative standing – like cars, jewelry, or expensive clothes (Frank and Heffetz, 2011). If one individual buys a new car to improve her status, others' well-being is negatively affected as their relative positions will *ceteris paribus* decline. Status consumption therefore imposes negative externalities, and people devote inefficiently high shares of their budgets to conspicuous or status goods while spending "too little" on other, non-status items. Public provision of inconspicuous goods might serve as an instrument to correct such under-consumption: by providing a certain level of a private good and financing it through incomes taxes, governments can alter how individuals spend their incomes and force them to consume less of status goods and more of non-status goods. Available empirical evidence views most publicly provided goods as rather less important for status reasons (Alpizar et al., 2005; Solnick and Hemenway, 2005; Charles et al., 2009). Concerns for relative standing may therefore provide a normative rationale for the public provision of private goods such as health care, basic education, or nutrition.

Finally, many government-provided goods possess characteristics of an investment good – in the sense that they involve up front costs but sizeable parts of their benefits arise in the future. For example, investments in education require considerable current expenses in exchange for an increased prospective earnings potential. Likewise, taking preventive medication, routine check ups, medical advice, or a proper diet mainly affect one's health status in the long-run. When deciding how much to consume or invest, people with present-biased preferences have a tendency to take immediate rewards and to avoid immediate costs, and attach lower weight to delayed benefits than they do from a more distant or long-run perspective. Due to such self-control problems, individuals' actual consumption choices may be against their own long-run interests. Thus, the presence of a present-bias might justify corrective government interventions like public provision.

## 1.2 Structure and summary of basic results

The thesis contributes both to the politico-economic and normative literature on public provision and includes three further chapters. Each chapter consist of a self-contained article. Chapter 2 is co-authored with Tobias König, Wissenschaftszentrum für Sozialforschung (WZB) and Humboldt University Berlin, and Andreas Wagener, Institute of Social Policy, Leibniz University Hannover. We introduce status concerns into a standard political-economy framework for the dual provision of private goods (see, e.g., Epple and Romano, 1996a). The government provides a tax-financed private good whose provision level is decided upon by majority voting. Individuals can take up the publicly provided level or opt out of the public system and buy the private good on a market. In addition to utility from consumption, both alternatives confer a certain social image or reputation on their consumers. The image utility from being (observed to be) a public system or market user is endogenous and positively depends on the average income in the group of individuals who make the same choice. We show that the difference in image utilities between private and public alternatives is positive. Intuitively, individuals who buy the private good on the market are on average richer. In addition, the social image of individuals out of and in the public system, respectively, increases with higher public provision: raising the provision level drives more people into the public system, and the average income of both groups rises; choosing the public alternative loses stigma, consuming on markets becomes even more prestigious or elitist. If this effect is sufficiently strong, even individuals who consume the private good on markets may support public provision or its expansion. Such a positive willingness to pay among non-users cannot arise when consumers only care for aspects of quality or prices. We find that image concerns can substantially affect the properties of majority voting equilibria. First, the set of possible political structures is richer; in addition to standard ends-against-the-middle and median income earner type equilibria, a novel ends-against-the-ends structure can arise. In such an equilibrium, the richer "ends" of individuals in and out of the public system (the "lower middle class" and the "rich") vote against an equal-sized coalition of individuals from the lower ends (the "poor" and the "upper middle class"). Second, private goods may be publicly provided in a majority voting equilibrium even though only a minority consumes the public alternative. Finally, individuals who buy the private good on markets may enjoy lower consumption levels than those consuming in the public system. Concerns for social image can therefore help to explain why housing, nutritional assistance, or transport are often publicly provided although a large majority consumes them in the market, and why private schools or private transport sometimes offer a lower quality of education or commuting than their public counterparts (see, e.g., Figlio and Stone, 1999).

Chapter 3 is co-authored with Tobias König. We formalize the idea that status concerns provide an efficiency rationale for the public provision private goods. Status is modeled as a preference for relative consumption, i.e., people care about how their own consumption compares to that of others. We consider a framework with two types of individuals ("rich" and "poor") who have preferences over a non-positional good for which only absolute consumption matters, and a positional good whose absolute as well as relative consumption is important. We find that under relatively mild conditions on status preferences, the introduction of a public provision scheme can always achieve Pareto improvements over a laissez-faire without government intervention. In line with recent empirical evidence, these conditions hold if (i) individuals - at least to some degree – compare themselves with members of similar groups ("within-group" comparisons) and (ii) if people consume more of the positional good if others' consumption levels rise ("keeping-up with the Joneses"). To underscore the efficiency-enhancing potential of public provision, we also study the case where a linear tax on the positional good is available. In particular, we identify simple necessary and sufficient conditions such that public provision can be Pareto-improving even if the positional tax is optimally chosen. Moreover, we show that a combination of public provision and a consumption tax can achieve Pareto efficient allocations – which is, in general, impossible if only a single instrument is available. However, under specific circumstances, public provision can implement efficient allocations alone, and therefore even dominate the taxation of the positional good.

In Chapter 4, I study how governments should intervene in markets for education, health care or old-age consumption when individuals have present-biased preferences. In particular, I consider the relative merits of two different policy instruments: price subsidies and public provision. This is motivated by the observation that many governments not only publicly provide but also subsidize private goods by lowering market prices. In Germany, for example, 30 percent of tuition fees for private schools can be deducted from parents' taxable income. Likewise, a number of countries offers tax benefits for participants in private pension schemes. But if public provision and price subsidies are simultaneously available, any government intending to address people's self-control problems faces the question of whether it should use only one instrument or a combination of both. I consider a dynamic economy with two private goods. One good is standard while the other requires immediate expenses but yields utility in the future. Individuals have present-biased preferences and may differ in gross incomes. I show that public provision strictly dominates price subsidies if people only differ in the intensity of their present-bias. When people have different biases and incomes, price subsidies and public provision can be complements. In the special case with two income types, this happens if and only if the present-bias is stronger among the poor. In the reverse case, the two policy instruments are substitutes: optimal policies include either a subsidy or public provision.

# Chapter 2

## The Public Provision of Private Goods with Image Concerns[1]

## 2.1 Introduction

When choosing from different varieties of goods, consumers not only care about prices and qualities but are also concerned with how they are perceived by others. Different varieties, brands, or labels of a good confer on their consumers different levels of distinction, prestige, esteem, or even stigma. These social evaluations are often associated with the characteristics – say, income, wealth, status, or education – of typical users of the varieties. Individuals pay attention to the perceptions and reputational embedding of their consumption and, by making appropriate choices, seek to display or improve their social standing (see, e.g., Veblen, 1899[1994]; Leibenstein, 1950; Frank, 1985a; Bagwell and Bernheim, 1996).

Image concerns, i.e., the desire to portray and communicate oneself in a socially positive way, not only affect the choices between the different varieties of marketed goods. They also matter for the choice between goods provided by governments and privately supplied alternatives. Governments indeed provide important (private) goods and services to their citizens – like schooling, health care, social housing, public transportation, and nutritional assistance – that have close substitutes commercially supplied on markets. In substantial domains of life, citizen-consumers can decide whether to consume the

[1]Earlier versions of the chapter were presented at the 2011 Workshop on Public Economics (Bremen, Germany), the 2012 CESifo Area Conference on Public Sector Economics (Munich, Germany), the 2012 MPI Econ Workshop (Bonn, Germany), the 17th Spring Meeting of Young Economists (Mannheim, Germany), the 68th Congress of the International Institute of Public Finance (Dresden, Germany), the 27th Congress of the European Economic Association (Malaga, Spain), the 2012 Annual Congress of the German Economic Association (Göttingen, Germany) and at the 7th Nordic Behavioral and Experimental Economics Conference (Bergen, Norway).

publicly provided variety or purchase the good on markets – and the choice between both options may, in addition to aspects of price, quality, and convenience, be shaped by intangible social and reputational concerns (Ireland, 1994). For instance, sending one's children to a private rather than to a public school often caters for needs of distinction, elitism, and classiness, and may conspicuously signal higher status, greater wealth, or refined tastes (see, e.g., Postlewaite, 1998; Levy and Razin, 2015). Similar effects prevail with kindergartens or health care (Mumtaz et al., 2013). Conversely, publicly provided varieties of a good are sometimes stigmatized, as with social housing, nutritional assistance (food stamps), and public transportation (see Steg, 2005, or Litman, 2009, on "bus stigma" in the U.S. or the UK). Then, people might choose not to take up the public option out of the fear of being stereotyped as unsuccessful, idle, or morally weak (see, *inter alia*, Moffitt, 1983; Besley and Coate, 1992).

A dual provision scheme with a choice between a publicly provided good and a private alternative differs markedly from selecting among varieties on markets. First, the provision levels for publicly provided goods are determined politically and not, as on markets, by price mechanisms. In democratic regimes, where majority voting is prominent (Epple and Romano, 1996a; Luelfesmann and Myers, 2011), all citizens – including the non-users – have a political say in decisions on public provision. Second, government-provided goods, which are typically made available to citizens free of charge or at subsidized prices and at a uniform level, are tax-financed. Individuals who opt out of the public system are still obliged to contribute, with their taxes, to financing public provision. *Ceteris paribus*, the public system can be more generous, in terms of per-capita provision levels, the fewer people choose the public option. Conversely, for those in the private system, extensive public provision and high taxes appear more and more undesirable.

If only the consumption of goods and services matters, each individual just weighs the (possibly zero) benefit from higher provision levels against the additional tax burden. Image concerns may considerably deflect political preferences for public provision. Different levels of public provision and taxes attract different people into the public system, affecting the social image both of public system and market users. If, for instance, a higher public provision level lures more people into the public system, this may reduce the stigma from choosing the publicly provided good. It may also raise social image rents from consuming in the private system, as such a choice now becomes more select. In this example, image concerns generate, both with users and non-users of the public option, a positive willingness to pay for (more) public provision – which might be stronger than, and in opposite direction to, standard consumption motives. Generally, the endogenous social feedback will change individuals' willingness to pay for the publicly provided good, and hence, the political economy of public provision, compared to a model where individuals only care for aspects of qualities and prices.

In this paper, we study the public provision of private goods under majority voting when individuals have concerns for their social image. We build on the standard political-economy framework for the dual provision of private goods, as proposed by Epple and Romano (1996a): the government provides a tax-financed private good whose provision level is decided upon by majority voting. Individuals can choose between taking up the publicly provided good and opting out of the public system and buying their individually most desired level in the market. To this framework, we add endogenous image concerns in the spirit of Corneo and Jeanne (1997) and Bénabou and Tirole (2011): individuals care about their social image, which other people deduce from their consumption choice. Specifically, consumption choices allow for inferences about income, and the ascription of higher incomes confers on individuals higher social (or self-)esteem. The image value of being (observed to be) a public or a private user is endogenous and, with a feedback loop, positively depends on the average income in the group of individuals that make the same choice. In our model, the endogenous "image gap" between individuals out of and in the public system is positive, and the image values of both groups rise if the level of public provision increases (Proposition 2.1).

When image concerns prevail, the citizenry is, at any given level of public provision, partitioned into (at most) four groups with respect to their political interests: (1) individuals who consume in the public system but would oppose a further increase in the public provision level; (2) individuals in the public system who would favor a higher public provision level; (3) individuals in the private system who would oppose higher public provision; and (4) individuals in the private system who are in favor of expanding public provision. The crucial group, compared to a situation where only consumption levels matter, is group 4. It can emerge since an expansion of the public system may make the private system more prestigious or elitist, thereby fueling the image concerns of outsiders. Such a positive willingness to pay among non-users of public provision cannot arise with standard consumption motives only.

A majority voting equilibrium is characterized by a provision level that exactly balances the numbers of supporters and of opponents to an increase in public provision: the unions of groups 1 and 3 and of groups 2 and 4 must each contain half of the population (see Lemma 2.1). Since some of the groups may be empty, this is compatible with a wide array of equilibrium configurations (see Propositions 2.2, 2.3 and 2.6).

This seemingly technical observation should be contrasted with the equilibrium predictions of standard dual provision scenarios (Epple and Romano, 1996a). With pure consumption concerns, group 4 does not exist: nobody in the private system would ever want to pay for more public provision. If, as assumed in Epple and Romano (1996a), groups 1 to 3 are ordered in the form of ascending income brackets a majority voting equilibrium has an "ends-against-the-middle" structure, with a coalition between "the poor" (group 1) and "the rich" (group 3) against middle-income earners (group 2). Image concerns – and with them, the potential emergence of group 4 – produce alternative equilibria (Proposition 2.3). E.g., a novel "ends-against-the-ends" equilibrium type can arise where the very rich (group 4) coalesce with the lower middle class (group 2) that also wishes to expand public provision for reasons of reduced stigma on those who take the publicly provided good. Conversely, the very poor and the upper middle class (groups 1 and 3) vouch for a retrenchment of public provision.

Image concerns can help to understand several puzzling observations in dual provision schemes for private goods. First, if individuals only care for consumption, a private good is publicly provided only if the majority of individuals actually takes up the public option (Epple and Romano, 1996a; Lindbeck et al., 1999). However, in several countries goods and services like housing, nutritional assistance and sometimes transport are government-provided although a large majority purchases them in the market (Currie, 2006). With image concerns, such a situation can arise as an equilibrium: the desire for distinction generates a positive benefit of public provision among its non-users (see Proposition 2.4). Second, if only consumption utilities matter, individuals who exit the public system will always choose a higher consumption level than the one supplied by the government. This prediction is in conflict, e.g., with the observation that private schools or private transport sometimes offer a lower quality of education or commuting than their public counterparts (see, e.g., Figlio and Stone, 1999). Similarly, eligible households who do not take up in-kind programs such as Food Stamps or Medicare in the U.S. forego substantial economic benefits while constraining themselves to lower qualities of food or medical care than offered in public programs (Currie and Gahvari, 2008). Such behavior might be driven by image concerns: seeking to gain in distinction or avoid stigma, individuals may substitute consumption for improved social status (see Proposition 2.5). Third, if only material concerns mattered, the rich would always oppose redistribution, both in cash or via tax-financed in-kind programs. With image concerns, dual provision schemes may appear attractive for the better-off: opting out provides a vehicle to display status and reap gains in image utility.

Our paper complements a growing literature on image and status concerns in consumer demand (e.g., Ireland, 1994; Pesendorfer, 1995; Bagwell and Bernheim, 1996; Glazer and Konrad, 1996; Corneo and Jeanne, 1997; Hopkins and Kornienko, 2004; Moav and Neeman, 2012) and suppliers' reactions to consumers' desires for distinction (Rayo, 2013; Friedrichsen, 2015; Vikander, 2015). These papers deal with the provision of goods in markets; to our knowledge, image concerns have not yet been analyzed in a dual provision, political economy model.[2] The literature on the political economy of status concerns is scarce. Corneo and Grüner (2000) study majority voting over redistribution when individuals want to signal status. By equalizing the distribution of disposable incomes, redistribution makes consumption lose its signaling capacity, thus reducing the political support for redistribution. In our framework, more redistribution (via public provision) increases the scope for distinction among the rich: public provision keeps the poor in the public system, thus enhancing the classiness of private alternatives. Levy and Razin (2015) argue that an expansion of the public sector and the attending reduction in income inequality softens the pressure (for the rich) to engage in costly signaling, e.g., through private schooling. Our approach differs in that in-kind redistribution is a vehicle for signaling, rather than a remedy against it: taxpayers forego public provision exactly because this gives them higher distinctiveness. On the flip side, categorizing consumers in the public system into a low-status group is related to the stigma from participation in welfare schemes, discussed in Lindbeck et al. (1999). Reduction in stigma then would strengthen the interest of individuals inside the public system to expand the system and broaden its membership. In our approach, the strive for distinction among those outside the public system reinforces the political clout for public provision (see Proposition 2.1).

The rest of this paper is organized as follows: Section 2.2 presents the model. Section 2.3 analyses the properties of endogenous image utilities. Section 2.4 discusses technical aspects of majority voting equilibria. Their political and economic features are described in Section 2.5. Section 2.6 generalizes our findings, and Section 2.7 concludes. All proofs

[2]Status motives are, however, discussed in other areas of public policy, including commodity taxation, regulation and income taxation. See, e.g., Truyts (2012) and the references therein.

are relegated to Appendix 2.A.

## 2.2 The model

### 2.2.1 Framework

**General:** The economy is populated by a continuum of individuals with measure one. Individuals differ in their exogenous gross incomes $y$. Incomes in the population are distributed according to a continuous cumulative distribution function $F(\cdot)$ with support on $\mathcal{Y} = [\underline{y}, \bar{y}] \subset (0, \infty)$. By $y_{med} = F^{-1}(1/2)$ and $Y = \int_{\mathcal{Y}} y \mathrm{d}F(y)$ we denote, respectively, median and average income in the economy. Subsets of the population will be abbreviated by capital letter $H$, possibly adorned by sub- or superscripts; the attending measure will be indicated by lower-case $h$. I.e., for $H \subseteq \mathcal{Y}$ we have $h = \int_{y \in H} \mathrm{d}F(y)$.

**Goods and their provision:** There are two private goods, denoted by $x$ and $c$. Good $c$, which serves as the numéraire, is a standard consumption good, exclusively provided by markets. Good $x$, which is also a rival good, is costlessly provided by the government at an equal per-capita level $\bar{x}$ to all individuals. As an alternative to consuming $\bar{x}$, individuals can opt out of public provision and buy their desired level on the market. Public and private sector consumption are mutually exclusive: individuals cannot supplement or diminish the publicly provided quantity via additional purchases or sales on the private market.

To finance public provision, the government levies a proportional income tax at rate $t$. Everybody has to pay the tax, irrespective of whether he consumes $\bar{x}$ or opts out. The production technology of good $x$ is linear and identical in the public and the private sector: one unit of the numéraire can be transformed into one unit of $x$. We correspondingly normalize the market price of good $x$ to one.

**Preferences:** All individuals have identical preferences. They derive utility from the consumption of goods $x$ and $c$, represented by a smooth, strictly increasing and strictly quasi-concave utility function $u(x, c)$. We assume that both goods are essential.[3]

In addition to consumption utility, the decision to consume good $x$ in the public system or to buy it on the market imparts to individuals a certain social reputation or (self-)

[3]Formally, $u(0, c_1) < u(x, c_2)$ for all $x, c_1, c_2 > 0$ and $u(x_1, 0) < u(x_2, c)$ for all $c, x_1, x_2 > 0$.

image. Both choices generate non-pecuniary “image rents”, depending on their social perceptions. Let $P_a$ denote the image value of consumption choice $a$, where we define variable $a$ to take value 0 for an individual who consumes the publicly provided $\bar{x}$ and 1 if she buys good $x$ on the market. In summary, preferences are given by

$$U(x, c, a) = u(x, c) + \beta \cdot P_a, \tag{2.1}$$

where scalar $\beta \geq 0$ measures the weight of image concerns. The additive separability between $u(x, c)$ and $P_a$ in (2.1) allows for a clear distinction between consumption and image utility. It implies that the two utility components do not directly interfere with one another.

The values $P_0$ and $P_1$ represent the feelings like prestige, distinction, or stigma associated with consumption choices for $x$. They are assumed to arise as the individual is, or perceives himself to be, socially equated with a typical consumer who makes the same choices as he does. Image values $P_a$ of consumption choices are endogenous in the economy, varying with the partition of individuals into consumers of the publicly provided level $\bar{x}$ and buyers on markets. Specifically, we assume that both $P_0$ and $P_1$ increase in the average incomes in the population subgroup they represent:

$$P_a = \mathbf{E}(y|a). \tag{2.2}$$

Here, $\mathbf{E}(\cdot|a)$ is the expectations operator, conditional on the choice $a \in \{0, 1\}$. The linearity of $U$ in $\mathbf{E}(y|a)$ is chosen for simplicity, reflecting a constant marginal relevance of image concerns.

**Interpretation:** Preferences (2.1) and (2.2) capture various social or individual perceptions of choosing between public provision and market purchases. One interpretation, akin to Corneo and Jeanne (1997) and Bénabou and Tirole (2011), is income or status signaling: choices between publicly provided and private options – such as sending one’s child to a private rather than to a public school or kindergarten or commuting by private car rather than by public transport – often involve a sorting according to incomes or wealth. Consumption decisions that are observable for social peers, thus, might easily communicate otherwise private information about one’s income or wealth (Charles et al., 2009; Heffetz, 2011). This may be beneficial if, e.g., higher incomes are associated with a higher social rank. It may also generate stigma if, e.g., public schemes for transportation, housing, or health insurance turn out to be the "poor man's schemes" and are associated with failure, idleness, or other stereotypes of low-income earners (Simanis, 1970).

Preferences (2.1) and (2.2) may also lend to an interpretation in terms of social norms. A public provision scheme redistributes from users to non-users of publicly provided goods (since taxes must be paid by everybody). Then, $P_0 - P_1$ can be interpreted as the image value of being a net contributor rather than a beneficiary from the public budget. If positive, $P_0 - P_1$ reflects so-called welfare stigma: living off welfare benefits is seen, from one's own or from society's view, as normatively inferior to contributing to the common good. Via (2.2), $P_0 - P_1$ will turn out to decrease in the number of public sector users, i.e., in the take-up rate of public provision.[4] Such endogenous welfare stigma – the psychological costs of welfare take-up decrease the more common such behaviour is – has been discussed frequently in social policy (Currie, 2003; Lindbeck et al., 1999).

**Sequence of events:** The model proceeds in three stages with the following chronology: in the first stage, a policy $(t, \bar{x})$ with an income tax rate $t$ and an expenditure level $\bar{x}$ is selected by majority voting (political equilibrium). In the second stage, each individual decides whether to consume good $x$ in the public system or to purchase it on the market, taking $(t, \bar{x})$ as given (decision over $a$). In the third stage, given $a$ and $(t, \bar{x})$, individuals spend their after-tax incomes to maximize utility (individual consumption choices). We solve the model by backward induction.

### 2.2.2 Consumption and system choice

**Stage 3:** Given $(t, \bar{x})$, an individual with gross income $y$ who opts out of public provision ($a = 1$) purchases the (unique) bundle $(x, c) > (0, 0)$ that maximizes $u(x, c)$ subject to the budget constraint $c + x = y(1 - t)$. Let $x^* = x^*(y(1 - t))$ and $c^* = c^*(y(1 - t)) = y(1 - t) - x^*(\cdot)$ be the Marshallian demand functions for goods $x$ and $c$. By the separability of (2.1), $x^*$ and $c^*$ are independent of image utility. The indirect utility from consumption is given by

$$v(y(1-t)) := u(x^*, y(1-t) - x^*).$$

[4] Lindbeck et al. (1999) directly write the number of welfare users in the utility function of welfare users. Our modeling can be interpreted as a micro-foundation of this approach.

If the individual consumes the publicly provided amount $\bar{x}$ (i.e., if $a = 0$), she spends her entire net income $y(1-t)$ on good $c$. Consumption utility then amounts to $u(\bar{x}, y(1-t))$.

**Stage 2:** Anticipating the decisions in Stage 3, an individual chooses $a = 1$ over $a = 0$ whenever the difference in image utilities between the consumption options is large enough to compensate for differences in consumption utility, i.e., if:

$$\beta \cdot (P_1 - P_0) \geq u(\bar{x}, y(1-t)) - v(y(1-t)). \tag{2.3}$$

For given image values $P_1$ and $P_0$ and policy $(t, \bar{x})$, condition (2.3) partitions the population into the set of individuals who consume in the private system ($a = 1$) and of those who choose the publicly provided level ($a = 0$). We denote these groups by

$$\begin{aligned} \tilde{H}^{out}(P_1 - P_0, t, \bar{x}) &:= \{y \in \mathcal{Y} | \text{ Condition (2.3) holds}\}, \\ \tilde{H}^{in}(P_1 - P_0, t, \bar{x}) &:= \mathcal{Y} \setminus \tilde{H}^{out}(P_1 - P_0, t, \bar{x}). \end{aligned}$$

**Consistent expectations.** We require that expectations (and hence image values) are consistent with actions. Formally, at any policy $(t, \bar{x})$, image values $(P_1, P_0)$ must satisfy:

$$P_1 = \int_{y \in \tilde{H}^{out}(P_1 - P_0, t, \bar{x})} y \mathrm{d}F(y) \quad \text{and} \quad P_0 = \int_{y \in \tilde{H}^{in}(P_1 - P_0, t, \bar{x})} y \mathrm{d}F(y). \tag{2.4}$$

Denote the sets of individuals in and outside of the public system under consistent expectations by $\tilde{H}^{in}(t, \bar{x})$ and $\tilde{H}^{out}(t, \bar{x})$. We assume that their measures are continuously differentiable in $t$ and $\bar{x}$.

### 2.2.3 Balanced budget

In stage 1, a policy $(t, \bar{x})$ is selected by majority vote. We restrict the analysis to *feasible* policies, which both balance the government budget and involve consistent expectations. Formally, a policy $(t, \bar{x})$ is *feasible* if

$$t \cdot Y = \bar{x} \cdot \int_{y \in \tilde{H}^{in}(t, \bar{x})} \mathrm{d}F(y). \tag{2.5}$$

By the Implicit Function Theorem, (2.5) defines the provision level $\bar{x}$ as a continuously differentiable function of the tax rate $t$, i.e.,

$$\bar{x} = x(t). \tag{2.6}$$

Obviously, $x(0) = 0$. Without much loss in generality, we assume that all positive tax rates $t > 0$ that we consider go along with positive public provision levels $x(t) > 0$. By (2.6), the policy space is one-dimensional, with the tax rate $t$ as the remaining policy variable. Henceforth, we denote by

$$H^{in}(t) := \tilde{H}^{in}(t, x(t)) \quad \text{and} \quad H^{out}(t) := \tilde{H}^{out}(t, x(t))$$

the sets of individuals in and outside the public system. At feasible policy $(t, x(t))$, the image values of consuming in and outside the public system are given by

$$\begin{aligned} P^{in}(t) &:= P_0|_{(2.4),(2.5)} = \mathbf{E}(y|y \in H^{in}(t)) \quad \text{and} \\ P^{out}(t) &:= P_1|_{(2.4),(2.5)} = \mathbf{E}(y|y \in H^{out}(t)). \end{aligned}$$

## 2.3 Political preferences and image concerns

### 2.3.1 Preferences

We now define indirect individual preferences over policies, which will determine voting behavior. Given a tax rate $t$, denote by

$$\begin{aligned} V^{in}(t, y) &:= u(x(t), y(1-t)) + \beta P^{in}(t, x(t)), \\ V^{out}(t, y) &:= v(y(1-t)) + \beta P^{out}(t, x(t))) \quad \text{and} \\ V(t, y) &:= \max\{V^{out}(t, y), V^{in}(t, y)\} \end{aligned} \tag{2.7}$$

the indirect utility levels of an individual with income $y$ in the public system ($V^{in}$), out of the public system ($V^{out}$), and when choosing the better of the two ($V$). Observe that $V^{in}$, $V^{out}$, and $V$ incorporate utility-maximizing behavior of all other individuals, the partition of the population into $H^{in}$ and $H^{out}$, and the government budget constraint. Functions $V^{out}$, $V^{in}$, and $V$ are continuous and differentiable in $(t, y)$ with the exception that $V(\cdot, y)$ has a (zero-measure) non-differentiability when $V^{in}(t, y) = V^{out}(t, y)$.
Utilities $V^{out}$ and $V^{in}$ in (2.7) depend on the policy variable $t$ in two ways. First,

taxation affects consumption utilities $u(x(t), y(1-t))$ and $v(y(1-t))$. This effect is present in the absence of image concerns as well. Second, changes in $t$ alter the composition of public and private sector users and, thus, the image utilities $P^{in}$ and $P^{out}$ of individuals in and outside of the public system. These social feedback effects translate into additional motives to favor or oppose public provision, thereby shaping the coalition structure in voting equilibria (see below).

### 2.3.2 Assumptions on preferences

In the following, we will make a series of assumptions that ensure two standard features of models of dual provision (see, e.g., Epple and Romano, 1996a; Luelfesmann and Myers, 2011). Specifically, we require that (i) image concerns do not offset (monotone) sorting across incomes and (ii) that higher tax rates *ceteris paribus* make the public system more attractive.[5] Our first assumption requires that image concerns are not overriding consumption concerns at extreme tax rates:

**Assumption 2.1** *For every* $y \in \mathcal{Y}$, $V^{out}(0, y) - V^{in}(0, y) > 0$ *and* $V^{out}(1, y) - V^{in}(1, y) < 0$.

I.e., for very low tax rates and public provision levels, the public system is unattractive such that everybody buys good $x$ in the market, and for sufficiently high tax rate, everybody will consume $x$ in the public system and spend the entire net income on good $c$. In Epple and Romano (1996a) or Luelfesmann and Myers (2011), Assumption 2.1 is ensured by the strict quasi-concavity of $u$ and the essential-good property of $c$ and $x$.

Our second assumption says that opting out of the public system becomes more unattractive (or less attractive) the larger the tax rate and, hence, the public provision level:

**Assumption 2.2** *For all* $t$ *and* $y$, $\partial[V^{out}(t, y) - V^{in}(t, y)]/\partial t < 0$.

Together with Assumption 2.1, Assumption 2.2 implies that for every income level $y \in \mathcal{Y}$ there exists a unique tax rate $\hat{t}$ such that $V^{in}(t, y) \geq V^{out}(t, y)$ for all $t \geq \hat{t}(y)$. I.e., an individual with income $y$ stays in [opts out of] the public system whenever $t \geq \hat{t}(y)$ [$t < \hat{t}(y)$].

[5] All assumptions are phrased in terms of $V^{in}(t, y)$ and $V^{out}(t, y)$, and thus, are combined requirements on direct preferences ($u$), image concerns ($\beta P_a$), the distribution function ($F(y)$), and their interplay through (2.4) and (2.6).

The third assumption ensures that, if a person is in [out of] the public system, then so are all poorer [richer] persons: for any $t$, if $V^{in}(t, y') = V^{out}(t, y')$ at some income $y'$, then $V^{in}(t, y) \geq V^{out}(t, y)$ for all lower incomes $y$ and $V^{out}(t, y) \geq V^{in}(t, y)$ for all higher incomes $y$.

**Assumption 2.3** *For any $t$, $V^{out}(t, y) - V^{in}(t, y)$ strictly increases in $y$.*

### 2.3.3 Implications for image concerns

Assumptions 2.1 to 2.3 shape image utilities in the aggregate. First, by Assumptions 2.2 and 2.3 the tax rate of indifference increases in income: for all $y$,

$$\hat{t}'(y) > 0. \tag{2.8}$$

Intuitively, when individuals get richer, a higher tax rate (likewise, a higher public provision level) is needed to keep them consuming $x$ in the public system. From (2.8), $\hat{t}(y)$ can be inverted; we denote its inverse by $\hat{y}(t)$ with $\hat{y}'(t) > 0$. For a given tax rate $t$, there exists an income threshold $\hat{y}(t)$ such that individuals with incomes below [above] $\hat{y}$ stay in [out of] the public system; this threshold is higher for higher tax rates.
As a consequence, the sets of individuals in and out of the public system are, at every feasible policy $t$, the income brackets below and above $\hat{y}(t)$:

$$H^{in}(t) = [\underline{y}, \hat{y}(t)] \quad \text{and} \quad H^{out}(t) = (\hat{y}(t), \bar{y}]. \tag{2.9}$$

Consequently, the image values ascribed to consuming good $x$ in and out of the public system are the average incomes below and above the income threshold $\hat{y}$:

$$P^{in}(t) = \mathbf{E}(y|y \leq \hat{y}(t)) \quad \text{and} \quad P^{out}(t) = \mathbf{E}(y|y \geq \hat{y}(t)). \tag{2.10}$$

Image utilities have the following properties:

**Proposition 2.1** *Under Assumptions 2.1 to 2.3, $P^{out}(t) > P^{in}(t)$ for all $t \in (0, 1)$. Moreover,*

$$\frac{dP^{out}(t)}{dt} > 0 \quad \textit{and} \quad \frac{dP^{in}(t)}{dt} > 0. \tag{2.11}$$

Individuals who purchase good $x$ in the market enjoy higher image utility than users in the public system: by income sorting, individuals in $H^{out}(t)$ are richer. Moreover, image utilities $P^{in}$ and $P^{out}$ both increase in the tax rate. Higher tax rates attract richer individuals into the public system, raising average incomes both in and out of the public system. Consuming good $x$ in the public system loses stigma, buying it on markets becomes even more select.

The monotonicity of image utilities in (2.11) has important implications for political preferences $V^{out}$ and $V^{in}$. For individuals out of the public system, consumption utility $v(y(1-t))$ strictly decreases in the tax rate. In the absence of image concerns, individuals in the private system, thus, will always be in favor of cutting back tax and expenditure levels: for $\beta = 0$, $\partial V^{out}/\partial t < 0$. By contrast, image utilities entail a benefit from higher taxes ($dP^{out}/dt > 0$). If these image effects are strong enough, they can override the reduction in consumption utility and turn $\partial V^{out}(t)/\partial t$ positive. Hence, their image concerns can also make non-users support public provision or its expansion. For those who consume $\bar{x}$ in the public system, a higher tax rate involves a trade-off in consumption utility $u(x(t), y(1-t))$: it means a higher provision level $x(t)$ but comes at the cost of reducing the consumption level of the other good, $c = y(1-t)$. Depending on which effect dominates (which may vary with income and the prevailing tax rate), their materialistic concerns can lead consumers in the public system to favor expanding or cutting back public provision. Image concerns $P^{in}(t)$ add a marginal benefit from higher tax rates, *ceteris paribus* leading to stronger support for (or lower reluctance against) more public provision amongst users. However, if tax rates get large, the deterioration in consumption utility due to the low level of good $c$ will override the benefits from higher tax rates.

## 2.4 Majority voting equilibria

### 2.4.1 Definition and description

As usual, a majority voting equilibrium (MVE) is defined as a feasible tax rate that beats every other feasible tax rate in pairwise majority comparison:

**Definition 2.1** *A tax rate $t^*$ is a majority voting equilibrium (MVE) if*

*(i) the attending expenditure level balances the government budget: $\bar{x}^* = x(t^*)$, and*

*(ii) at least half of the population prefers, with respect to $V(t, y)$, policy $t^*$ to any other $(t, \bar{x})$ with $\bar{x} = x(t)$.*

*A MVE is called interior if and only if* $t^* > 0$ *and, consequently,* $x(t^*) > 0$*. A MVE has dual provision if both* $H^{out}(t^*) \neq \emptyset$ *and* $H^{in}(t^*) \neq \emptyset$*.*[6]

We now provide a technical result that will be instrumental for our characterization of MVE below. Denote by

$$\begin{aligned} t^{in}(y) &:= \arg\max_{t \geq \hat{t}(y)} V^{in}(t, y), \\ t^{out}(y) &:= \arg\max_{t \leq \hat{t}(y)} V^{out}(t, y) \quad \text{and} \\ t(y) &:= \arg\max_{t} V(t, y) \end{aligned}$$

the most preferred tax rates for a person with income $y$, provided, respectively, she consumes the publicly provided option, opts out of public provision, or chooses optimally. Given $t$, define the following four (not necessarily non-empty) subsets of individuals:

$$\begin{aligned} H_-^{in}(t) &= H^{in}(t) \cap \{y | t^{in}(y) < t\}, \\ H_+^{in}(t) &= H^{in}(t) \cap \{y | t^{in}(y) \geq t\}, \\ H_-^{out}(t) &= H^{out}(t) \cap \{y | t^{out}(y) < t\}, \\ H_+^{out}(t) &= H^{out}(t) \cap \{y | t^{out}(y) \geq t\}. \end{aligned}$$

The first two sets split the group of individuals in the public system into those who would favor a (marginal) reduction of the tax rate ($H_-^{in}(t)$) and those who would like to see the tax rate to be increased ($H_+^{in}(t)$). Sets $H_-^{out}(t)$ and $H_+^{out}(t)$ do the same for the group of individuals who opt of public provision at $t$. The sets $H_-^{in}(t)$ to $H_+^{out}(t)$ correspond to the groups 1 to 4 informally described in the Introduction.

**Lemma 2.1** *Suppose Assumptions 2.1 to 2.3 hold. In an interior MVE with dual provision the following holds true:*

$$h_-^{in}(t^*) + h_-^{out}(t^*) = \frac{1}{2} = h_+^{in}(t^*) + h_+^{out}(t^*).$$

Lemma 2.1 states that there are two opposing coalitions in an interior MVE: those who advocate slightly higher tax rates, $H_+^{out}(t) \cup H_+^{in}(t)$, and those who advocate slightly lower tax rates, $H_-^{out}(t) \cup H_-^{in}(t)$. Both coalitions encompass half of the population and, thus, exactly offset one another.

[6] MVE without dual provision are uninteresting: if none is in the public system at a MVE, $t^*$ is trivially zero. If everybody is in the public system, the median income earner is decisive, i.e., $t^* = \arg\max V(t, y_{med})$.

Lemma 2.1 does not imply though that all four groups $H_-^{in}(t)$ to $H_+^{out}(t)$ are non-empty. Depending on whether any of them and, if so, which are empty, various interesting coalitional structures arise.

### 2.4.2 Preference profiles

The properties of a MVE generically depend on the distribution of favorite policies or, what is the same, political preferences $V$ over voters' types (i.e., incomes). In dual provision systems, political preferences need not be single-peaked over the policy space $t \in [0,1]$ (see, e.g., Stiglitz, 1974; Epple and Romano, 1996a; Barbera and Moreno, 2011; Luelfesmann and Myers, 2011). Hence, the median-voter theorem typically does not apply. In the following, we will endow the collection of preferences $\{V(\cdot, y)\}_{y\in\mathcal{Y}}$ with (more) structure. We choose assumptions such as to nest the standard dual provision scenario by Epple and Romano (1996a) as the special case when image concerns are absent ($\beta = 0$).

We first exclude the uninteresting case that the richest persons choose to consume in the public system at the median income earner's favorite policy:

**Assumption 2.4** $H^{out}(t^{in}(y_{med})) \neq \emptyset$.

Assumption 2.4 appears as (A.6) in Epple and Romano (1996a). If it does not hold, the median income earner is the decisive voter in an interior MVE.

We next assume that, for all $y$, the most-preferred tax rates $t^{out}(y)$ and $t^{in}(y)$ are unique and that, *within* a consumption mode, no individual is ever indifferent between two tax rates on the same side of his utility peak:

**Assumption 2.5** *For every $y \in \mathcal{Y}$, $V^{in}(t, y)$ and $V^{out}(t, y)$ are single-peaked in $t$.*

Single-peakedness of $V^{in}$ is also assumed in Luelfesmann and Myers (2011). Since consumption for individuals outside the public system strictly decreases in $t$, single-peakedness of $V^{out}$ is trivially satisfied in Epple and Romano (1996a) and Luelfesmann and Myers (2011). It needs, however, to be explicitly assumed in the presence of image concerns. Single-peakedness of both $V^{out}$ and $V^{in}$ does not imply single-peakedness of their upper envelope $V$ (see Barbera and Moreno, 2011). Naturally, however, $t(y) \in \{t^{out}(y), t^{in}(y)\}$. By construction, $t^{out}(y) \leq \hat{t}(y) \leq t^{in}(y)$.

The final two assumptions govern the distribution of favorite tax rates $t^{out}(y)$ and $t^{in}(y)$ across incomes. Starting with Epple and Romano (1996a), dual provision models typically assume that favorite tax rates vary (weakly) monotonically in incomes. For consumers in the public system, two scenarios are discussed: preferred tax rates $t^{in}(y)$ increase or decrease in income. As only the former scenario generates interesting results in the absence of image concerns, we choose to study it in the first place (the latter scenario is covered by the general analysis of Proposition 2.6 below). Hence, we impose[7]

**Assumption 2.6** *For all $y > y'$,*

$$\frac{\partial V^{in}(t,y')}{\partial t} \geq 0 \quad \Longrightarrow \quad \frac{\partial V^{in}(t,y)}{\partial t} > 0.$$

Assumption 2.6 and the maximum property of $t^{in}(y)$ entail that favorite tax rates are weakly ordered in incomes: $t^{in}(y) \geq [\leq]\, t^{in}(y')$ for all $y > [<]\, y'$. These inequalities are strict whenever $t^{in}(y')$ is an interior maximum characterized by $\partial V^{in}(t,y')/\partial t = 0$. I.e., richer individuals in the public system prefer higher tax rates. Keeping with the logic of Assumption 2.6 also for those in the private system, we suggest

**Assumption 2.7** *For all $y > y'$,*

$$\frac{\partial V^{out}(t,y')}{\partial t} \geq 0 \quad \Longrightarrow \quad \frac{\partial V^{out}(t,y)}{\partial t} > 0.$$

Assumption 2.7 implies that $t^{out}$ (weakly) increases with income. It is always satisfied in the absence of image concerns (where $\partial V^{out}/\partial t < 0$ and $t^{out} = 0$ for all $y$). Economically, Assumption 2.7 conveys that the gain in image utility or social distinction from making the private system more elitist is increasing in income. In an instrumental interpretation of image concerns, Assumption 2.7 conveys that the utility gain from mixing with the rich is complementary to income − which is a frequent assumption in matching models (see, e.g., Levy and Razin, 2015). The monotonicity in Assumptions 2.6 and 2.7 only refers to preferences within each consumption system; no assumption is made on marginal utility across systems.

By Assumptions 2.1 to 2.7, subsets $H_-^{in}$ through $H_+^{out}$ are disjoint intervals that partition $\mathcal{Y}$ into ascending income brackets. In Section 6 we show that the richness of coalitional structures in our model does not hinge upon Assumptions 2.6 and 2.7.

[7] Epple and Romano (1996a) couch their analysis in terms of marginal willingnesses to pay. Assumption 2.6 then corresponds to their so-called SRI case. As in Luelfesmann and Myers (2011) and Glomm and Ravikumar (1998), we equivalently phrase assumptions in terms of indirect utilities.

## 2.5 Political and economic features of MVE

### 2.5.1 Equilibrium configurations

To appreciate Proposition 2.3 below, we briefly consider the case without image concerns: $\beta = 0$. There, $t^{out}(y) = 0$ for all $y$ and $H_+^{out}(t)$ is empty.

**Proposition 2.2 (Epple and Romano, 1996a, Prop. 3)** *Suppose that $\beta = 0$ and that Assumptions 2.1 to 2.7 hold. An interior MVE with dual provision is of the "ends-against-the-middle" type, i.e.,*

- *there exist incomes $y_1, y_2$ with $y_1 < y_{med} \leq y_2$ such that $t^* = t^{in}(y_1) = \hat{t}(y_2) < t^{in}(y_{med})$;*
- *$h_+^{out}(t^*) = 0$ and $h_-^{in}(t^*) + h_-^{out}(t^*) = h_+^{in}(t^*) = 1/2$.*

The ends-against-the-middle MVE in Proposition 2.2 derives its name from its coalitional structure (see Epple and Romano, 1996a, for a full discussion). At $t^*$, the upper and lower "ends" of the income distribution – given by the income brackets $[\underline{y}, y_1)$ in the public system and $(y_2, \bar{y}]$ out of the public system (with $y_1 < y_2$) – would favor a lower tax rate than $t^*$ while an equally large group of individuals with intermediate incomes (the "middle") would like to see the tax rate increased. Both the (combined) ends and the middle constitute half of the population. Consequently, the median income earner belongs to the middle group and consumes in the public system but the equilibrium tax rate and provision level are lower than he would ideally have them.[8]
The next result reports that with image concerns the coalitional structures that can prevail in a MVE are richer:

**Proposition 2.3** *Suppose that $\beta > 0$ and that Assumptions 2.1 to 2.7 hold. If an interior MVE $t^* > 0$ is such that the median income earner consumes in the public system (i.e., if $y_{med} \in H^{in}(t^*)$), then the MVE is of either of the following types:*

*(A) "Ends-against-the-middle".*

*(B) "Ends-against-the-ends":*

[8] In a limiting case, $H_-^{in}$ can be empty. Then $h_+^{in} = h_-^{out} = 1/2$ and $y_2 = y_{med}$. At the expense of some notational clutter (formally, the definition of $y_1$ in (2.14) would have to be adjusted), this could still be modelled as a degenerate ends-against-the-middle equilibrium. We will not pursue this here; also see case (D.1) in Proposition 2.4.

- *there exist incomes* $y_1 < y_2 < y_3$ *such* $t^* = t^{in}(y_1) = \hat{t}(y_2) = t^{out}(y_3) < t^{in}(y_{med})$*;*
- *all four of* $h_-^{in}(t^*), h_-^{out}(t^*), h_+^{in}(t^*), h_+^{out}(t^*)$ *are positive.*

*(C) "Median income earner":*

- $t^* = t^{in}(y_{med})$*;*
- $h_-^{out}(t^*) = 0$ *and* $h_-^{in}(t^*) = h_+^{in}(t^*) + h_+^{out}(t^*) = 1/2$*.*

Item (A) of Proposition 2.3 states that ends-against-the-middle MVE can arise also in the presence of image concerns. Items (B) and (C) show that image concerns considerably widen the variety of possible MVE configurations. A novel equilibrium type is the *ends-against-the-ends* MVE (B). Here, all four types of political preference, each held within a separate income bracket, are relevant. Individuals in the bottom end of the income distribution ($H_-^{in} = [\underline{y}, y_1]$) consume the publicly provided option but would prefer less of it. They coalesce with the "upper middle" class ($H_-^{out} = [y_2, y_3)$) who consumes $x$ in the private system and would like to see public provision curtailed. These two groups of opponents to public sector expansion are politically offset by two groups of proponents: the "lower middle" class ($H_+^{in} = [y_1, y_2)$) that consumes the publicly provided option and the "rich" ($H_+^{out} = [y_3, \bar{y}]$) who are out of the public system but would still favor its expansion since that would boost their image utility from belonging to the more select group of outsiders. As in the end-against-the-middle case, the MVE tax is strictly lower than the median-income earner's favorite tax rate: $t^* < t^{in}(y_{med})$. Item (C) shows that also *median income earner*-type MVE can arise: $t^* = t^{in}(y_{med})$. There, *everybody* outside the public system prefers to have a higher tax rate (i.e., $H_-^{out}$ is empty) and, in that, coalesces with the higher-income earners in the public system. Only the poor in the public system, $H_-^{in} = [\underline{y}, y_1)$, object to tax rises. Since political preferences are monotonically aligned with incomes, the MVE is the median income earner's preferred tax rate.

Equilibrium types (B) and (C) involve individuals who buy good $x$ on their own but still like to see public provision expanded (group $H_+^{out}$ is non-empty). In the absence of image concerns, such a preference cannot prevail out of the public system, and proponents for public system expansion can only come from among public system users. Image concerns provide a motive for non-users to endorse higher public provision: the benefit from the attending increase in social status exceeds the deterioration in consumption utility. By

Assumption 2.7, this motive is stronger for richer individuals. If it is strong enough only for some of the non-users of public provision (viz., the richest group), then the MVE will be of the *ends-against-the-ends* type (B); for some (not-so-rich) consumers outside the public system image gains do not override their materialistic preference for lower tax rates ($H_{-}^{out} \neq \emptyset$). If everybody outside the public system wants higher taxes (i.e., if image concerns are sufficiently strong), the median income earner MVE (C) will prevail. Finally, if image concerns are weak, we are back in the ends-against-the-middle MVE of Proposition 2.2 (case (A)).

Proposition 2.3 characterizes the possible types of interior MVE. It does not provide any conditions for the emergence of a certain type (or for MVE existence). Such conditions require, in a complex way, restrictions on consumption utilities, the income distribution, and the strength of image concerns. Appendix 2.A.5 demonstrates by a worked example with CES consumption utility and a Weibull-type distribution of incomes that all MVE types of Proposition 2.3 can indeed arise (in that example, by letting the strength, $\beta$, of image concerns vary alone). The example also illustrates that one cannot expect any monotonic relation between $\beta$ and the tax rate or provision level in a MVE.

### 2.5.2 Public provision for a minority

In the absence of image concerns ($\beta = 0$), those who opt out of the public system always prefer a zero tax rate and provision level to any other policy. Hence, positive public provision can only arise as a MVE if a majority of individuals actually uses the public supply. By Assumption 2.3, this requires the median income earner to be in the public system: if $\beta = 0$ and $t^* > 0$, then $y_{med} \in H^{in}(t^*)$ (Epple and Romano, 1996a, Prop. 2). This feature is certainly met for publicly provided goods such as schools or public health services. In the cases of housing or (in some countries or cities) transportation, goods *are* actually publicly provided while the majority chooses private alternatives. Image concerns can help to explain such puzzles: even individuals who buy good $x$ on markets might benefit from the existence of a public sector as it enhances their image utility from opting out. If this effect is strong enough, the decisive voter is willing to provide a positive level of $\bar{x}$ even if he himself relies on the private market. Majority voting then leads to the provision of a good that the majority does not consume:

**Proposition 2.4** *Suppose that Assumptions 2.1 to 2.7 hold. If $\beta > 0$, the median income earner can be out of the public system in an interior MVE: it is possible that*

$t^* > 0$ *and* $y_{med} \notin H^{in}(t^*)$.

Proposition 2.4 is verified in Appendix 2.A.6. There we provide an example with CES consumption utilities and a uniform income distribution that features the majority in the population supporting public provision while only a minority actually uses it.
One can show that the potential coalitional structures in a MVE where the median income earner opts out of public provision are the same as those listed in Proposition 2.3.[9] An ends-against-the-ends equilibrium has exactly the same coalitional structure as before: the upper [lower] ends in the public and private system forge a political alliance for [against] a higher tax rate. The ends-against-the-middle equilibrium, however, remarkably differs: when the majority is outside the public system, the rich and the poor "ends" still form a political coalition but they now would vote for an *expansion* of public provision (the middle class, buying in the market, favors tax cuts). In that sense, image concerns can completely upset political preferences, relative to the standard framework.

### 2.5.3 Public vs. private provision levels

In the absence of image concerns, everybody who opts out of public provision in a MVE purchases a higher level (or quality) of good $x$ than the level $\bar{x}$ provided in the public system.[10] This prediction is, however, not always in line with empirical evidence. For instance, private schools do not always offer better quality than their public counterparts (Figlio and Stone, 1999). Likewise, in many cities those who commute by car accept significantly longer travel times or higher stress levels through congestion, although convenient and quick public transport systems may exist. Conversely, many eligible households do not take up in-kind transfer programs like Food Stamps or Medicare in the U.S., although they can privately afford only a lower level of food or medical care than offered in the public program (Currie and Gahvari, 2008).
Image concerns might explain why individuals choose a lower level of good $x$ than $\bar{x}$.[11] In exchange for the gains in image utility from opting out of the public system some individuals might be willing to accept lower consumption levels of *both* goods when buying them on the market (for private car use, see Steg, 2005, or Litman, 2009). Con-

[9] The proof of this assertion follows, *mutatis mutandis*, the same logic as the proof of Proposition 2.3 and is omitted.

[10] For $\beta = 0$, individuals who opt out of the public system are characterized by $u(x^*(y(1-t)), c^*(y(1-t))) > u(\bar{x}, y(1-t))$. As $c^* = y(1-t) - x^* < y(1-t)$, this can only hold if $x^*(y(1-t)) > \bar{x}$.

[11] For education, alternative explanations have been put forward by, e.g., Figlio and Stone (1999), Martinez-Mora (2006) or Brunello and Rocco (2008).

versely, the stigmatization of public in-kind provision (relative to private consumption) can make people avoid program take-up and sacrifice better consumption (Moffitt, 1983; Besley and Coate, 1992).

Exiting public provision happens when

$$\beta \left[P^{out}(t) - P^{in}(t)\right] > u(x(t), y(1-t)) - u(x^*, c^*). \tag{2.12}$$

For $\beta > 0$, the gain in image utility from opting out is strictly positive (see Proposition 2.1). Hence, image concerns will induce some individuals to opt out of public provision even if that diminishes their consumption utility, i.e., if $u(x^*, c^*) < u(x(t), y(1-t))$. Regardless of image concerns, opting out of public provision always goes along with a reduced consumption of good $c$ (since $c^* < y(1-t)$). In the presence of image concerns (and only then), also the consumption of good $x$ may be lower for those who opt out of public provision:

**Proposition 2.5** *In an interior MVE $t^*$ with image concerns, it is possible that $x^*(y(1-t^*)) < x(t^*)$ for some, and possibly even for all, $y \in H^{out}(t^*)$.*

The example in Appendix 2.A.6 can again serve as **proof**. There, the consumption level of $x$ for *everybody* who opts out of the public mode is lower in a MVE than the publicly provided level $\bar{x}$. In less extreme scenarios (not constructed here), the private consumption of good $x$ will be lower than the public level only for some of those who opt out of public provision. Due to the normality of good $x$, this applies to individuals close to the threshold of opting out (i.e., for whom (2.12) just holds). By Assumptions 2.1 to 2.3, these are the poorest among those who, in their desire to avoid stigma or to mingle with the rich, opt out of public provision.

## 2.6 Generalization

Political preferences and, consequently, the coalitional structure in a MVE depend on the interaction of direct preferences $u(x, c)$, the income distribution $F$, and the specifics of image concerns. If political preferences exhibit income monotonicity in the sense of Assumptions 2.6 and 2.7, the specific equilibrium predictions in Propositions 2.3 (and 2.4) arise.

Independently of income monotonicity, a population that harbours image concerns can, at any feasible policy, be partitioned with respect to political preferences into the four subgroups $H_-^{in}$ to $H_+^{out}$ introduced in Section 2.4.1 (not all types of political preference need to materialize in a MVE). As in an interior MVE the groups of opponents and supporters of further tax increases are of equal size (Lemma 2.1), the coalitional structures in an interior, dual provision MVE with image concerns can only be of limited variety, even without Assumptions 2.6 and 2.7:[12]

**Proposition 2.6** *Suppose that $\beta > 0$ and that Assumptions 2.1 to 2.5 hold. An interior MVE $t^*$ with dual provision has either of the following coalitional structures:*

*(A.1)* $H^{out}(t^*) = H_-^{out}(t^*)$ *and* $h^{in} > 1/2$*: the majority of the population is in the public system; all individuals out of the public system (and some inside) prefer a lower tax rate than* $t^*$*.*

*(A.2)* $H^{in}(t^*) = H_+^{in}(t^*)$ *and* $h^{out} > 1/2$*: the majority of the population is out of the public system; all individuals in the public system (and some outside) prefer a higher tax rate than* $t^*$*.*

*(B) None of* $H_-^{in}(t^*)$ *to* $H_+^{out}(t^*)$ *is empty: both among those in the public system and among those outside there are opponents and supporters of an increase in the tax rate beyond* $t^*$*.*

*(C.1)* $H^{out}(t^*) = H_+^{out}(t^*)$ *and* $h^{in} > 1/2$*: the majority of the population is in the public system; all individuals outside of the public system (and some inside) prefer a higher tax rate than* $t^*$*.*

*(C.2)* $H^{in}(t^*) = H_-^{in}(t^*)$ *and* $h^{out} > 1/2$*: the majority of the population is outside the public system; all individuals in the public system (and some outside) prefer a lower tax rate than* $t^*$*.*

Assumptions 2.6 and 2.7 turn the subgroups $H_-^{in}$ to $H_+^{out}$ into ascending income brackets. The equilibrium types end-against-the-middle, ends-against-the-ends, and median income earner characterized in Proposition 2.3 under the proviso that $y_{med} \in H^{in}(t^*)$ then correspond to (A.1), (B), and (C.1) in Proposition 2.6. Similarly, the MVE configurations mentioned below Proposition 2.4 for $y_{med} \notin H^{in}(t^*)$ are represented by (A.2), (B), and (C.2) in Proposition 2.6.

[12]Proposition 2.6 does not list two uninteresting knife-edge cases; see the proof.

As a first application of Proposition 2.6, consider a scenario where the inequalities in Assumptions 2.6 and 2.7 are reversed. Economically, this means that poorer consumers of the publicly provided good are less reluctant to finance an expansion than richer consumers; among those who opt out of public provision, the willingness to bear tax increases that entice others back into the public system is lower for the very rich than for the moderately rich. Without image concerns, the MVE is always of the median income earner-type (Epple and Romano, 1996a, SDI-case). In the presence of image concerns, however, a similarly rich assortment of possible MVE configurations as in Proposition 2.3 emerges.[13] In particular, both ends-against-the-middle and ends-against-the-ends equilibria can arise.

Proposition 2.6 is also applicable without uniform monotonicity assumptions on preferred tax rates. The precise structure of a MVE then depends on the distribution of the signs of $\partial V^{in}(t,y)/\partial t$ and $\partial V^{out}(t,y)/\partial t$ over $\mathcal{Y}$, allowing for an even finer segmentation of the population and manifold coalitional structures. In summary, image concerns substantially enlarge the set of political equilibrium patterns.

Finally, Proposition 2.6 also shows that the feature that, with image concerns, public provision might arise in a MVE although a majority does not take up the public program, is compatible with several types of coalitional structures.[14]

## 2.7 Conclusion

Governments provide goods to their citizens that are at least partly private in nature: education, housing, transport, health services etc. In democratic regimes, such provision must find support among a majority of citizens[15] whose political preferences for or against (a larger volume of) public provision are shaped by individualistic cost-benefit deliberations as well as by social motives. Our analysis demonstrates that social motives, here exemplified by endogenous image concerns, can substantially affect the political and economic properties of voting equilibria.

Our paper contributes to a better understanding of redistributive policies (or the "welfare state") in various ways. First, a dual provision scheme potentially endows society

[13] Formally, reverting the inequalities in Assumptions 2.6 and 2.7 implies that both $t^{in}(y)$ and $t^{out}(y)$ decrease in $y$ (on their domains), inverting the income stratification in the subgroups of $H^{in}$ and $H^{out}$.

[14] Equilibria of type (B) in Proposition 2.6 can go along with both $h^{in} > 1/2$ or $h^{in} < 1/2$.

[15] From a social planner perspective, the public provision of private goods can also be motivated by externality or market failure arguments (see, e.g., Levy 2005).

with a screening mechanism: if the partitioning of the population into non-users and users of publicly provided goods runs parallel to a stratification into high- and low-status groups, image-concerned non-users may be willing to subsidize public provision although they do not materially benefit from it. Image concerns, thus, complement social motives such as altruism (Coate, 1995), concerns for equal opportunities (Gasparini and Pinto, 2006) or paternalistic preferences that also help to explain why, e.g., certain private goods are publicly provided though the majority does not take them up. Unlike image concerns, these types of social preferences cannot, however, accommodate for private consumption being at a lower level than public provision.

Second, every redistributive mechanism, whether in cash or in-kind, partitions the population into two social groups – beneficiaries and net contributors – and can, in principle, generate informative signals about an underlying status-bearing personal characteristic such as income, a strong work ethic etc. Our results, thus, generally apply for both cash and in-kind transfers. However, taking up or declining a publicly provided good (such as schools, housing, transport) is more openly visible than receiving (or even not-receiving) a monetary payment. In-kind transfers are more discriminatory than cash transfers (Besley and Coate, 1992), and image concerns will work to a stronger effect with in-kind programs. In fact, due to image concerns in-kind programs might survive in a political process where cash transfers fail. This observation may contribute to the ongoing debate why there is so much in-kind redistribution although cash transfers are available and economically superior (see, e.g., Currie and Gahvari, 2008).

Generally, in the presence of image concerns richer individuals are likely to favor, for a given level of redistribution, a discriminatory in-kind system whereas poorer individuals will support the more anonymous cash-transfer system. To study which system (or mixture of systems) will prevail in a majority voting equilibrium is a promising idea for future research.

# Appendix 2.A

## 2.A.1 Proof of Proposition 2.1

The fact that $P^{out} > P^{in}$ can directly be seen from (2.10): individuals in the private system are uniformly richer than in the public sector. Next calculate:

$$\frac{dP^{out}}{dt} = \frac{\mathrm{d}}{\mathrm{d}\hat{y}}\left(\frac{1}{1-F(\hat{y})}\int_{\hat{y}}^{\bar{y}} yf(y)\mathrm{d}y\right)\cdot\frac{d\hat{y}}{dt} = \frac{f(\hat{y})}{1-F(\hat{y})}\cdot(\mathbf{E}(y|y\geq\hat{y})-\hat{y})\cdot\frac{d\hat{y}}{dt} > 0,$$

since $d\hat{y}/dt$ is positive by (2.8). Likewise, one shows that

$$\frac{dP^{in}}{dt} = \frac{f(\hat{y})}{F(\hat{y})}\cdot(\hat{y}-\mathbf{E}(y|y\leq\hat{y}))\cdot\frac{d\hat{y}}{dt} > 0.$$

This proves (2.11). ■

## 2.A.2 Proof of Lemma 2.1

Note that $H^{in}_{-}(t)$ through $H^{out}_{+}(t)$ partition $\mathcal{Y}$ by construction. Hence, $h^{in}_{-} + h^{out}_{-} + h^{in}_{+} + h^{out}_{+} = 1$ for all $t$, where all measures are continuous. By the dual provision property, Assumptions 2.1 and 2.2, we have $t > \hat{t}(\underline{y})$. Any $t > \hat{t}(\underline{y})$ with $h^{out}_{-}(t) + h^{in}_{-}(t) > 1/2 > h^{out}_{+}(t) + h^{in}_{+}(t)$ can be defeated in majority vote against a suitably chosen, slightly lower tax rate; any $t > \hat{t}(\underline{y})$ with $h^{out}_{-}(t) + h^{in}_{-}(t) < 1/2 < h^{out}_{+}(t) + h^{in}_{+}(t)$ would lose against a slightly higher tax rate. Hence, only tax rates such that $h^{out}_{-}(t) + h^{in}_{-}(t) = h^{out}_{+}(t) + h^{in}_{+}(t) = 1/2$ can be MVE. ■

## 2.A.3 Proof of Proposition 2.2

Suppose that $\beta = 0$ and consider an interior MVE $t^* > 0$ with dual provision. Since, in the absence of image concerns, $h^{out}_{+}(t) = 0$ for all feasible $t$, Lemma 2.1 directly implies that the coalitional structure at $t^*$ must be such that $h^{in}_{-}(t^*) + h^{out}_{-}(t^*) = h^{in}_{+}(t^*) = 1/2$. Moreover, $y_{med} \in H^{in}(t^*)$ and, thus, $h^{in}(t^*) \geq 1/2 \geq h^{out}_{-}(t^*) > 0$, the latter due to dual provision. Moreover, a threshold $\hat{y}(t^*)$ exists such that $\hat{y}(t^*) \geq y_{med}$. It can be shown that for a MVE[16]

$$t^{in}(\underline{y}) \leq t^* < t^{in}(y_{med}) \tag{2.13}$$

[16] See Appendix 2.A.8 or Proposition 2 and Corollary 2 in Epple and Romano (1996a).

must hold. Since we require the majority to stay in the public system, all tax rates $t \leq \hat{t}(y_{med})$ can be ruled out as a MVE. Hence, a MVE $t^*$ must come from the interval $[\max\{\hat{t}(y_{med}), t^{in}(\underline{y})\}, t^{in}(y_{med}))$. It, thus, satisfies

$$\frac{\partial V^{in}(t^*, \underline{y})}{\partial t} \leq 0 < \frac{\partial V^{in}(t^*, y_{med})}{\partial t}.$$

By the continuity of the distribution $F(y)$ and Assumption 2.6 there exists

$$y_1 = \max\left\{ y \in [\underline{y}, y_{med}) \middle| \frac{\partial V^{in}(t^*, y)}{\partial t} \leq 0 \right\}. \tag{2.14}$$

Moreover, set $y_2 = \hat{y}(t^*)$. By Assumption 2.6 we then get that $H_-^{in}(t^*) = [\underline{y}, y_1)$, $H_+^{in}(t^*) = [y_1, y_2)$, and $H_-^{out} = [y_2, \bar{y}]$.[17] ■

### 2.A.4 Proof of Proposition 2.3

By similar lines of reasoning as in the proof of Proposition 2.2, it can be excluded that a MVE has $t^* > t^{in}(y_{med})$: in these cases $h_-^{in}$ would be strictly larger than 1/2, contradicting Lemma 2.1. Since we require the majority to stay in the public system, all tax rates $t < \hat{t}(y_{med})$ can be ruled out as a MVE. Should they exist, tax rates in $[\hat{t}(y_{med}), t^{in}(\underline{y}))$ can also be disregarded as MVE: they would have $h_+^{in} > 1/2$ by Assumption 2.6. Hence, an interior MVE $t^*$ with $y_{med} \in H^{in}(t^*)$ must lie in $[\max\{\hat{t}(y_{med}), t^{in}(\underline{y})\}, t^{in}(y_{med})]$. The dual provision property requires that $H^{out}(t^*)$ is non-empty. Thus, at most one of $h_+^{out}(t^*)$ and $h_-^{out}(t^*)$ can be zero. This gives rise to three cases:

(A) Assume that only $h_+^{out}(t^*)$ is zero. From Lemma 2.1, $h_+^{in}(t^*) = 1/2$ and $h_-^{in}(t^*) \geq 0$. By Assumptions 2.6 and 2.7, subsets $H_-^{in}(t^*)$, $H_+^{in}(t^*)$ and $H_-^{out}(t^*)$ partition $\mathcal{Y}$ into ascending income brackets, where the latter two subsets are separated by $\hat{y}(t^*) =: y_2$. Define $y_1$ as in (2.14). Recall that $t^{in}(\underline{y}) \leq t^{in}(y_1) < t^{in}(y_{med})$. Then a MVE has $t^* = t^{in}(y_1)$ and $H_-^{in}(t^*) = [\underline{y}, y_1)$, $H_+^{in}(t^*) = [y_1, y_2)$, and $H_-^{out}(t^*) = [y_2, \bar{y}]$, as in Proposition 2.2.[18]

(B) Assume that both $h_-^{out}(t^*)$ and $h_+^{out}(t^*)$ are non-zero. Dual provision and the fact

[17] For completeness, consider the case that Assumption 2.4 does not hold. Then $t^{in}(y_{med})$ is the only candidate for an interior MVE by the median-voter theorem: If $t > t^{in}(y_{med})$, then $V^{in}(t^{in}(y_{med}), y) > V^{in}(t, y)$ for all $y \leq y_{med}$ and for some $y > y_{med}$, a set with a measure larger than 1/2. If $t < t^{in}(y_{med})$, then $V^{in}(t^{in}(y_{med}), y) > V^{in}(t, y)$ for all $y \geq y_{med}$ and for some $y < y_{med}$, again a set of measure larger than 1/2.

[18] A limiting case, already reported in footnote 8, with $h_-^{in}(t^*) = 0$ and $h_+^{in}(t^*) = 1/2 = h_-^{out}(t^*)$ can also arise here.

that $t^* \geq \hat{t}(y_{med})$ imply that both $h_-^{in}(t^*)$ and $h_+^{in}(t^*)$ are strictly greater than zero, too.[19] Non-emptiness of all four subgroups $H_-^{in}(t^*), H_+^{in}(t^*), H_-^{out}(t^*)$ and $H_+^{out}(t^*)$ and income sorting from $H_-^{in}(t^*)$ to $H_+^{out}(t^*)$ implies that $t^* = t^{in}(y_1) = \hat{t}(y_2) = t^{out}(y_3)$ for three distinct $y_1 < y_2 < y_3$. Depending on whether $h^{in}$ is equal to or strictly greater than 1/2, $\hat{y}(t^*) = y_{med}$ or $\hat{y}(t^*) > y_{med}$ and $t^* = \hat{t}(y_{med})$ or $t^* > \hat{t}(y_{med})$. Moreover, since $h_-^{in}(t^*) < 1/2$, $t^* < t^{in}(y_{med})$.

(C) Assume that only $h_-^{out}(t^*)$ is zero. By Lemma 2.1, $h_-^{in}(t^*) = 1/2$ (and $h_+^{in} \geq 0$). Due to income sorting, $t^* = t(y_{med})$. If $h_+^{in} > 0$, $t^{out}(y_{med}) < t^* = t^{in}(y_{med})$ and $y_{med} < \hat{y}$. If $h_+^{in} = 0$, $t^* = t^{in}(y_{med}) = t^{out}(y_{med}) = \hat{t}(y_{med})$ and $y^* = y_{med} = \hat{y}$.

This gives rise to configurations (A) through (C), as claimed. ■

### 2.A.5 Example for Proposition 2.3

The simulations underlying this and all following other examples were done with the help of Mathematica. Source codes are available on request.
Assume that consumption preferences are represented by CES utility function

$$u(x, c) = \frac{1}{1-\gamma}\left(\alpha x^{1-\gamma} + (1-\alpha) c^{1-\gamma}\right), \tag{2.15}$$

with $\alpha = 0.01$ and $\gamma = 1.5$. Image utilities are defined as in (2.2). Incomes are distributed according to

$$F(y) = \begin{cases} 0 & y \leq \underline{y} \\ 1 - e^{-(y/\sigma)^\mu} & \underline{y} < y \leq y_a \\ 1 + e^{-(y_a/\sigma)^\mu} \frac{(y-\bar{y})}{(\bar{y}-y_a)} & y_a < y \leq \bar{y} \\ 1 & \text{otherwise.} \end{cases}$$

This piecewise distribution is Weibull on $[\underline{y}, y_a)$ and uniform on $[y_a, \bar{y}]$ (the piecewise specification ensures that the support of $F$ is bounded). Setting $\underline{y} = 0$, $y_a = 15$, $\mu = 0.35$ and $\sigma = 0.37$, this distribution is positively skewed with median $y_{med} = 0.13$ and mean $Y = 5.96$. It can be verified that Assumptions 2.1 to 2.7 are satisfied.

[19] With dual provision, $h_-^{in}(t^*)$ and $h_+^{in}(t^*)$ cannot be zero at the same time. If $h_-^{in}(t^*) \neq 0$ and $h_+^{in}(t^*) = 0$, then $h_+^{out}(t^*) = 1/2$ from Lemma 2.1. Since $h_-^{out}(t^*) \neq 0$ by assumption, $H^{out}$ would comprises more than 50 % of the population, including (from Assumptions 2.1 to 2.3), the median income earner. If $h_-^{in}(t^*) = 0$ and $h_+^{in}(t^*) \neq 0$, then $h_-^{out}(t^*) = 1/2$, which, together with $h_+^{out}(t^*) > 0$, contradicts $t^* \geq \hat{t}(y_{med})$ as well.

Table 2.1 reports the features of the MVE if we sequentially increase the strength of image concerns, represented by $\beta$. In particular, the MVE type changes from ends-against-the-middle to ends-against-the-ends to median income earner. The equilibrium

| $\beta$ | $t^*$ | $x(t^*)$ | $\hat{y}$ | MVE type |
|---|---|---|---|---|
| 0.000018 | 0.008 | 0.05 | 2.56 | ends-against-the-middle |
| 0.0002 | 0.010 | 0.07 | 2.68 | ends-against-the-ends |
| 0.001 | 0.012 | 0.08 | 1.90 | median income earner |

Tab. 2.1: MVE in Example 1

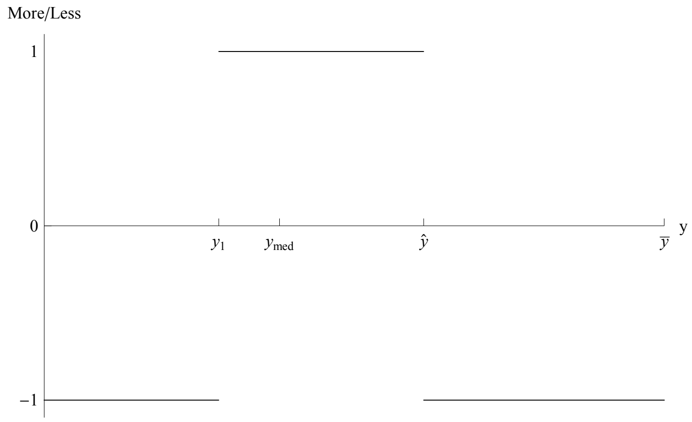
structures are visualized in Figure 2.1. The vertical axis in each panel depicts the sign of $\partial V(t^*, y)/\partial t$ for $y \in \mathcal{Y}$ and, thus, indicates whether the individual would prefer a higher (represented by value +1) or a lower tax rate (−1) than $t^*$. The union of the plus-groups and the union of the minus-groups form the political coalitions, $H_+^{in} \cup H_+^{out}$ and $H_-^{in} \cup H_-^{out}$. Income level $\hat{y}$ separates users outside and inside the public system. At the jumps, political preferences change. E.g., in Panel (A), individuals with incomes smaller than $y_s = 0.04$ or above $\hat{y} = 2.56$ prefer a lower tax rate while individuals with incomes between $y_s$ and $\hat{y}$ (including $y_{med}$) favor tax rates larger than $t^*$. This constitutes an end-against-the-middle configuration. Panels (B) and (C) depict ends-against-the-ends and median income earner configurations. In all panels, the plus- and the minus-groups each have measure 1/2 with respect to $F$. In Appendix 2.A.9 we verify that the $t^*$ reported in Table 2.1 indeed constitute MVE: they garner at least a 50%-majority in all binary comparisons against alternative feasible tax rates (including $t = 0$).

### 2.A.6 Example for Propositions 2.4 and 2.5

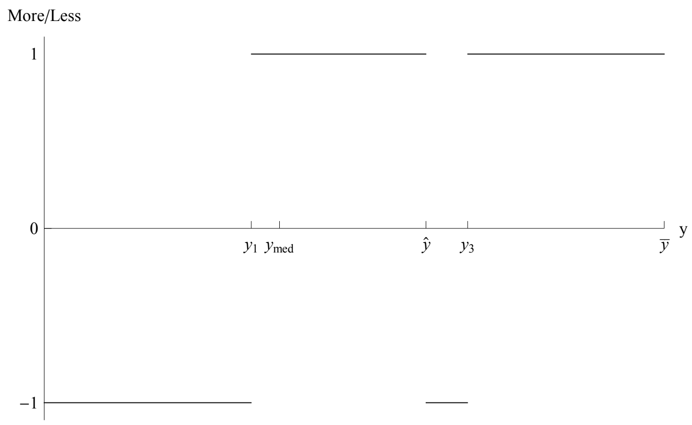
In this example we use CES consumption preferences (2.15) with parameters $\alpha = 0.1$ and $\gamma = 1.02$. Image utility has strength $\beta = 0.01$. Incomes are uniformly distributed on $[\underline{y}, \bar{y}] = [0, 100]$ such that $y_{med} = Y = 50$. The interior MVE is of the ends-against-the-middle type (see panel (i) in Figure 2.2). As panel (ii) demonstrates, it has two interesting features:

- The median income earner and, with him, a majority of individuals opt out of public supply at $t^*$ (cf. Proposition 2.4).

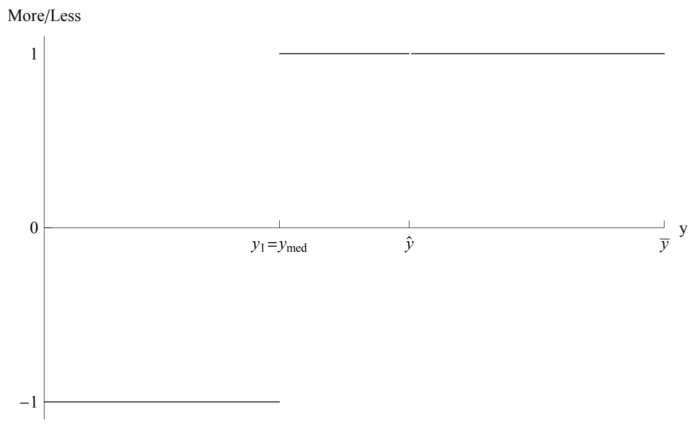
Fig. 2.1: Equilibrium configurations




(A): Ends-against-the-middle ($\beta = 0.00018$)



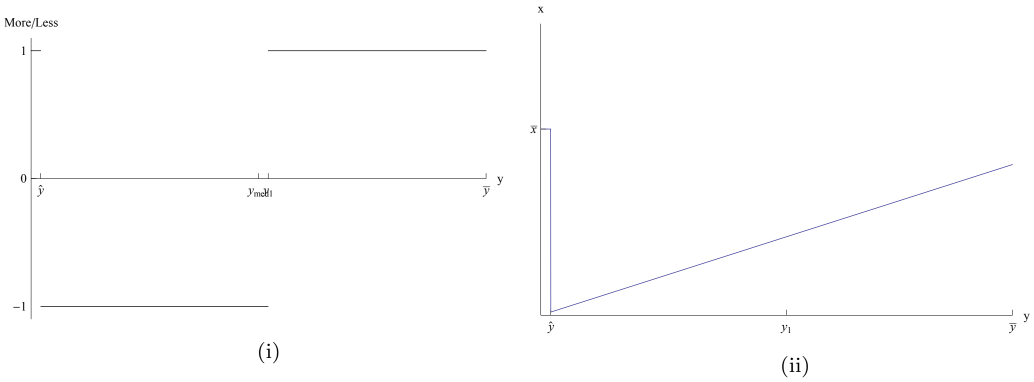

(B): Ends-against-the-ends ($\beta = 0.0002$)




(C): Median income earner ($\beta = 0.001$)

- Everybody outside the public system (including the median income earner) purchases less of good $x$ than $\bar{x} = x(t^*)$ (cf. Proposition 2.5).

Fig. 2.2: Median opts out and all choose less than $\bar{x}$




## 2.A.7 Proof of Proposition 2.6

By Assumptions 2.1 to 2.3, the sets $H^{in}(t)$ and $H^{out}(t)$ are, for any feasible tax rate $t$, non-empty intervals of $\mathcal{Y}$, separated by $\hat{y}$. Moreover, subsets $H_-^{in}(t)$ to $H_+^{out}(t)$ (some of which can be empty) partition $\mathcal{Y}$. Since $h_-^{in}(t^*) + h_-^{out}(t^*) = h_+^{in}(t^*) + h_+^{out}(t^*) = 1/2$ holds in any interior MVE (Lemma 2.1), at most two of $H_-^{in}$ to $H_+^{out}$ can be empty. Additionally, the dual provision property precludes that cases $H_-^{in} = H_+^{in} = \emptyset$ and $H_-^{out} = H_+^{out} = \emptyset$ can arise. Hence, the following seven cases are an exhaustive enumeration of what can happen in a MVE $t^*$:

(A.1) $H_+^{out}(t^*) = \emptyset$ (while all other subsets are non-empty): Then $h_+^{in} = 1/2$. Moreover, $h_-^{out} > 0$, $h_-^{in} > 0$, and both sum up to $1/2$. Moreover, $h^{in} > 1/2$, implying $y_{med} \in H^{in}$.

(A.2) $H_-^{in}(t^*) = \emptyset$: Then $h_-^{out} = 1/2$. Moreover, $h_+^{out} > 0$, $h_+^{in} > 0$, and both sum up to $1/2$. Moreover, $h^{out} > 1/2$, implying $y_{med} \in H^{out}$.

(B) None of $H_-^{in}$ to $H_+^{out}$ is empty at $t^*$: Then $t^* = t^{in}(y') = t^{out}(y'')$ for two distinct $y'' > y'$ with $y' \in H^{in}$ and $y'' \in H^{out}$ (two decisive voters). In the groups both of those in the public system and of those outside, there are individuals with who prefer higher as well as individuals who prefer lower tax rates than $t^*$.

(C.1) $H_-^{out}(t^*) = \emptyset$: Then $h_-^{in} = 1/2$. Moreover, $h_+^{out} > 0$, $h_+^{in} > 0$, and both sum up to $1/2$. Consequently, $h^{in} > 1/2$, implying $y_{med} \in H_{in}$.

(C.2) $H_+^{in}(t^*) = \emptyset$: Then $h_+^{out} = 1/2$, $h_-^{out} > 0$, $h_-^{in} > 0$, and $h^{out} > 1/2$, implying $y_{med} \in H^{out}$.

(D.1) $H_+^{in} = H_-^{out} = \emptyset$: Then $h_-^{in} = h_+^{out} = 1/2$. Hence, $t^{-1}(t^*) = y_{med}$. All individuals with below-median incomes are in the public system and would prefer a higher tax rate; all others are out and would prefer a lower tax rate.

(D.2) $H_-^{in} = H_+^{out} = \emptyset$: Then $h_+^{in} = h_-^{out} = 1/2$. Again, $t^{-1}(t^*) = y_{med}$. All individuals with below-median incomes are out of the public system and would prefer a lower tax rate; all others are in and would prefer a higher tax rate.

Equilibria (D.1) and (D.2) are uninteresting knife-edge cases, omitted from the proposition. ■

### 2.A.8 Proof of (2.13)

Clearly, any $t \leq \hat{t}(\underline{y})$ can be excluded as an interior MVE since it would be unanimously out-voted by any smaller $t$. More strictly, all $t_0 \in (\hat{t}(\underline{y}), t^{in}(\underline{y}))$ can also be excluded as an interior MVE. At any such $t_0$, $H^{in}(t_0) = [\underline{y}, y_0]$ with $y_0 = \hat{y}(t_0) > \underline{y}$ from Assumption 2.2.

- If $y_0 > y_{med}$, all individuals with $y \in [\underline{y}, y_0]$ would prefer a slightly higher $t$ to $t_0$; this is due to the fact that $t_0 < t^{in}(\underline{y}) < t^{in}(y)$ for all $y \in (\underline{y}, y_0]$ by Assumption 2.6. Since $y_0 > y_{med}$, we have $h^{in}(t_0) > 1/2$, and $t_0$ cannot be a MVE.

- If $y_0 < y_{med}$, then $h^{out}(t_0) > 1/2$ such that $t_0$ would lose a majority vote against any smaller $t$. Hence, $t_0$ again cannot be a MVE.

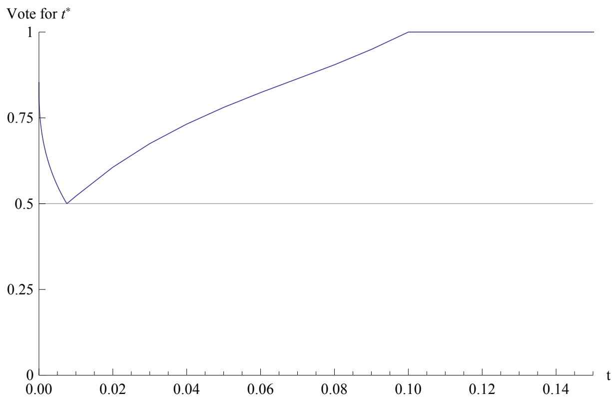
Finally consider $t^{in}(y_{med})$. By Assumption 2.6, $t^{in}(y_{med}) > t^{in}(\underline{y})$. Assumption 2.4 together with Assumptions 2.1 to 2.3 implies that there exists $y'$ such that $V^{out}(t^{in}(y_{med}), y) > V^{in}(t^{in}(y_{med}), y)$ for all $y > y'$. As a consequence, $t^{in}(y_{med})$ cannot be a MVE. It loses in majority comparison against all $t'$ smaller than, but suitably close to, $t^{in}(y_{med})$: everybody in $H^{out}(t')$ (which is non-empty since $H^{out}(t^{in}(y_{med})) \neq \emptyset$ and $t'$ is close to $t^{in}(y_{med})$) and everybody in $\{y | V^{in}(t', y) > V^{in}(t^{in}(y_{med}), y)\}$ prefer $t'$ to $t^{in}(y_{med})$. Since the measure of the latter set alone approaches $1/2$ for $t'$ suitably close to $t^{in}(y_{med})$, the two sets together have measure strictly larger than $1/2$.

Since a policy $t > t^{in}(y_{med})$ cannot be a MVE either (it would lose a majority vote against $t^{in}(y_{med})$), an interior MVE must satisfy (2.13). ■

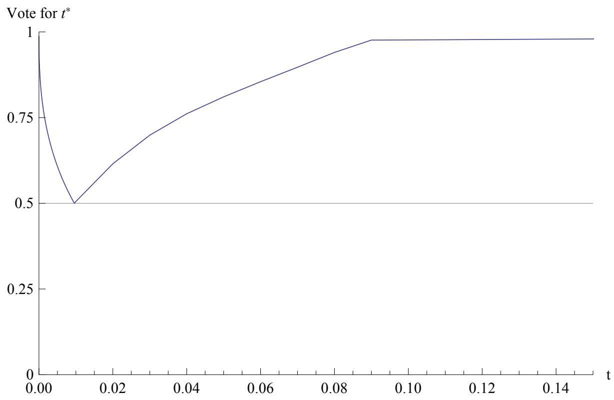
### 2.A.9 Verification of MVE for Example 2.A.5

To confirm that the tax rates $t^*$ reported in the example of Appendix 2.A.5 are indeed MVE, we let each of them run in pairwise majority comparison against all alternative feasible tax rates (including $t = 0$). Panels (A) to (C) in Figure 2.3 plot the shares of individuals preferring $t^*$ in pairwise comparison; for graphical reasons we only plot tax rates in a range from 0 to 0.15. As can be seen, the $t^*$ always garner more than 50 percent of the popular vote and are, thus, indeed MVE. In the example, equilibria gradually change from type (A) via (B) to (C) when the intensity of image concerns increases. This monotonicity is not general, though. For instance, for $\beta \geq 0.015$, the equilibrium again has the ends-against-the-ends structure (B). Likewise, the equilibrium values of tax rate $t^*$ and public provision level $\bar{x}(t^*)$ vary non-monotonically with $\beta$.

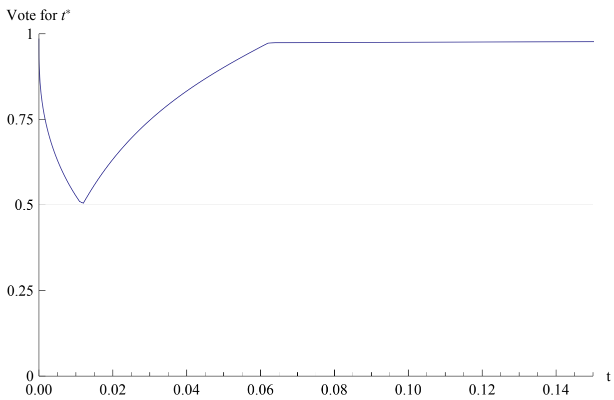
Fig. 2.3: Vote shares for $t^*$




(A): $\beta = 0.000018$




(B): $\beta = 0.0002$




(C): $\beta = 0.001$

# Chapter 3

## Relative Consumption and the Public Provision of Private Goods[1]

### 3.1 Introduction

In most countries, a significant share of the public budget is devoted to the provision of goods that are essentially private in nature – such as health care, education, child care, housing, and nutrition. The volume of public in-kind spending ranges from 5 to 20 percent of GDP in members of the OECD, and is still growing in many countries, both developed and developing (Bearse et al., 2000; Currie and Gahvari, 2008). Traditional economic theory offers no rationale for publicly providing specific private goods: as government-provided goods are typically also available on markets, replacing public provision with cash payments of equal value should (weakly) increase welfare, since then, people can choose consumption bundles that best suit their preferences.

In the past decades, several normative explanations for public provision have been put forward, including paternalism (Sandmo, 1983; Besley, 1988), motives of redistribution under informational constraints (Nichols and Zeckhauser, 1982; Blackorby and Donaldson, 1988; Besley and Coate, 1991), positive (production) externalities (Barr, 2012), and notions of equality of opportunity (Gasparini and Pinto, 2006). In this paper, we provide an alternative explanation: the public provision of private goods may serve as an instrument to correct inefficient consumer choices when people have concerns for relative consumption.

[1] An earlier version of the chapter is available as Number 510 of Hannover Economic Papers (HEP). Earlier versions were presented at 18th Spring Meeting of Young Economists (Aarhus, Denmark), the 14th Meeting of the Association for Public Economic Theory (Lisbon, Portugal), the 69th Congress of the International Institute of Public Finance (Taormina, Italy), the 28th Congress of the European Economic Association (Gothenburg, Sweden) and at the 2014 Annual Conference of the Royal Economic Society (Manchester, Great Britain).

While standard economic theory assumes that individuals only derive utility from goods and services *per se*, there is growing evidence in line with the idea that consumers' choices and satisfaction are also affected by the comparisons they make: individuals relate their own consumption to that of others and feel pleasure [discomfort] when they possess more [less] than their social peers.[2] For instance, people may enjoy their new car, but it feels even nicer when the car is bigger than that of one's friends, colleagues, or neighbors. Conversely, the Smiths may be no longer satisfied with their latest purchases if they see the Joneses enjoying a more lavish lifestyle – and engage in further spending races to try catching up (Dupor and Liu, 2003; Hopkins and Kornienko, 2004).

As argued by Hirsch (1976) and Frank (1985b), and supported by findings in the empirical literature on conspicuous consumption (see, e.g., Charles et al. 2009), relative consumption concerns are not equally important for all goods.[3] There are "positional" goods, whose value depends relatively strongly on how they compare with things consumed by others, and "non-positional" goods for which only one's own, absolute consumption matters.[4]

When people care for relative consumption, preferences are interdependent. If one individual consumes more of a positional good, others' relative consumption and well-being *ceteris paribus* decline. As people neglect that their positional spending imposes a harm on others, unregulated consumption choices are inefficient; people spend "too much" on positional goods while spending "too little" on other, non-positional items. Then, restricting consumer choices via public provision can be desirable: by providing a certain level of a non-positional good and financing it through incomes taxes, governments can alter how individuals spend their incomes and induce them to consume less of positional goods and more of non-positional goods.

We formalize this argument in a model with two private goods and two types of individ-

[2]The idea of relative consumption concerns traces (at least) back to Veblen (1899) and Duesenberry (1949). For recent evidence on the importance of relative consumption concerns, see, for example, Frey and Stutzer (2002), Bowles and Park (2005), Luttmer (2005), Solnick and Hemenway (2005), Carlsson et al. (2007), Clark et al. (2008), Charles et al. (2009), Clark and Senik (2010), Heffetz (2011), Frank and Heffetz (2011), Friehe and Mechtel (2014) and Roth (2014).

[3]See also Solnick and Hemenway (2005), Heffetz (2011) and Roth (2014).

[4]People might care about consumption positions for several reasons (Hopkins and Kornienko, 2004; Bilancini and Boncinelli, 2012). For example, to the extent that a favorable consumption position conveys a superior position in some underlying status scale (e.g., wealth or income), a preference for relative consumption may reflect a desire for high social status (Arrow and Dasgupta, 2009). Likewise, concerns for relative consumption may also originate from feelings of jealousy, envy or relative deprivation (Dupor and Liu, 2003). We do not presuppose one or another candidate motive. What is crucial to our analysis is the hypothesis that an individual suffers utility losses when others' consumption levels rise.

uals who differ in exogenous gross incomes ("rich" and "poor"). One good is positional, the other non-positional. In addition to the absolute consumption of both goods, individuals care about how their consumption of the positional good compares to an endogenous reference level – of which they dislike being behind and enjoy being ahead. Reference levels may differ across income groups and are modeled as a general, weakly increasing function of both groups' average positional spending. This formulation allows us to distinguish between different types of social comparisons discussed in the literature on social psychology and economics (Falk and Knell, 2004) – including upward, downward, or within-group comparisons.[5] We start our analysis with the public provision of *non-positional* goods, which is typically attributed to health care, old-age provision, nutrition, and basic education (Solnick and Hemenway, 2005; Charles et al. 2009; Heffetz, 2011). In particular, the government may provide a uniform amount of the non-positional good that is offered to all individuals free of charge and can be topped up by additional purchases on a market. To finance public provision, (lump-sum) income taxes can be levied.

We find that public provision can always achieve Pareto improvements over a laissez-faire without government intervention if at least one income group compares its positional good consumption with that of the poor – a condition satisfied by many specifications of reference functions such as average, within-group, or downward comparisons. To get an intuition, assume that the government sets the public provision level slightly above the amount consumed by the poor in the laissez-faire and raises income taxes by an equal-valued amount. This policy change does not alter the material utilities of both income types: as the rich had purchased a larger amount of the non-positional good anyway, their consumption choice is not affected; they take the publicly provided level and top it up through additional purchases. The poor are forced to marginally reduce their spending on the positional and to increase the consumption of the non-positional good, which – by the Envelope-Theorem – produces only negligible second-order effects. However, if at least one income type socially compares with the poor, the enforced reduction in positional spending of the poor decreases (some) individuals' reference levels, and thus, has positive first-order welfare effects.

We extend our simple model to several directions. In our basic set-up, social comparisons only have an effect on utility but not on consumption behavior. However, an

[5] While earlier contributions focus on comparisons with the economy-wide average positional consumption (see, e.g., Dupor and Liu, 2003), a growing body of work studies the policy implications of different comparison motives (see, for example, Micheletto, 2008; Eckerstorfer and Wendner, 2013).

individual's relative position may also affect her marginal propensity to consume. While the presence of such peer effects in consumption considerably complicates the analysis and can give rise to multiple equilibria, we derive a tractable sufficient condition for public provision to be always Pareto-improving. This condition not only requires that individuals compare themselves with the poor, but also that people consume more of the positional good if others' consumption levels rise, a property that is labeled "keeping-up with the Joneses" and considered the relevant case in the recent empirical literature on conspicuous consumption (see, e.g., Kuhn et al., 2011; Roth, 2014).

To underscore the efficiency-enhancing potential of public provision, we also study the case where a uniform tax on the positional good – a standard policy recommendation to address inefficiencies from status consumption – is available (Ireland, 1994; Hopkins and Kornienko, 2004; Micheletto, 2008; Eckerstorfer and Wendner, 2013). We provide simple necessary and sufficient conditions such that public provision can be Pareto-improving even if the tax is optimally chosen. Basically, there is room for public provision in all cases where the "marginal social harms" of positional good consumption differ across social groups. Moreover, we show that a combination of public provision and a consumption tax can achieve Pareto efficient allocations – which is, in general, impossible if only a single instrument is available. However, under specific circumstances, public provision might implement efficient allocations alone, and therefore even dominate the taxation of the positional good.

As certain publicly provided goods like higher education are sometimes considered as positional (Frank, 1985a), we finally discuss the role of the public provision of *positional* goods. Pareto improvements are then only possible in a provision system where additional purchases on top of the publicly provided level are infeasible. Intuitively, public provision could not distort individuals' positional good consumption downwards if they were allowed to buy additional units on markets.

The relevance of our analysis is supported by the growing evidence that concerns for relative standing play a crucial role in explaining individual consumption patterns and seem to be particularly important among lower income groups. For instance, using U.S. household consumption data, Charles et al. (2009) find that minorities (Blacks and Hispanics) spend more on conspicuous goods (jewelry, clothes) than do comparable Whites. These expenditure differences are associated with substantial diversions of resources from inconspicuous or non-positional goods such as education, health care, and food. Similar patterns hold for developing countries, where the poor devote relatively large fractions of their incomes on festivals, weddings, or funerals (Banerjee and Duflo, 2007; Case et al., 2008). Such behavior is often interpreted as satisfying needs to signal a high relative standing (Moav and Neemann, 2010, 2012), and, in most cases, financed by borrowing against the future (e.g., reduced old-age provision) or by diverting resources away from basic education or health prevention like mosquito nets, preventive drugs, or basic vaccination (Brown et al., 2011; Khamis et al., 2012; Moav and Neemann, 2012). Public provision is an effective instrument to correct such status-driven distortions in consumer choices: as long as demands for the non-positional and under-consumed goods are increasing in income, governments can separately target the poor while leaving richer individuals unaffected – an objective that a uniform consumption tax cannot achieve.

We contribute to the theoretical literature on the public provision of private goods or in-kind transfers (see Currie and Gahvari, 2008, for a survey). In contrast to the majority of existing explanations, we abstain from redistributive or paternalistic motives and provide a pure efficiency rationale. This is not to say that relative consumption concerns are the only (normative) justification for public provision; rather, we consider our rationale as complementary to the existing ones – and, given the importance of status concerns and social comparisons for consumption choices and individual well-being, as an empirically relevant additional explanation.

Our paper is also related to the literature studying the policy implications of concerns for relative standing. Earlier contributions mainly focus on price instruments – either in the form of consumption taxes and subsidies (Corneo and Jeanne, 1997; Hopkins and Kornienko, 2004; Micheletto, 2008; Truyts, 2012; Eckerstorfer and Wendner, 2013) or income taxation (e.g., Boskin and Sheshinski, 1978; Blomquist, 1993; Ireland, 1998, 2001; Aronsson and Johansson-Stenman, 2008, 2010). We complement existing studies by highlighting the usefulness of quantity instruments to cope with the inefficiencies arising from relative consumption concerns – a role that has been largely ignored in previous work. An important exception is Ireland (1994). Using a signaling framework, he develops the idea that public provision or in-kind spending is an instrument to reduce people's consumption of conspicuous goods. However, he does not perform any welfare analysis. We extend his study and show that public provision can indeed induce Pareto improvements. In addition, while Ireland (1994) only uses quasi-linear examples, our results apply for more general preference specifications. Finally, we provide a joint analysis of consumption taxes and public provision.

The paper is structured as follows. Section 3.2 introduces the theoretical framework and presents the economic problem. Section 3.3 illustrates the efficiency-enhancing potential of public provision. Peer effects in consumption are considered in Section 3.4. Section 3.5 studies the case where consumption taxes are available. Section 3.6 analyzes whether Pareto efficient allocations can be attained. Section 3.7 considers the public provision of positional goods, while the final section concludes. All proofs are relegated to Appendix 3.A.

## 3.2 The model

### 3.2.1 Framework

**General:** Consider an economy populated by a large number of individuals. Individuals can be of two types $i = 1, 2$ who differ in their endowments or gross incomes $y^i$, where $y^1 < y^2$. We henceforth label individuals of type 1 as "the poor" and of type 2 as "the rich". For simplicity, the number of each type is normalized to one. There are two private goods, denoted $c$ and $x$. Both goods are produced by perfectly competitive industries using a linear production technology and individuals' endowments $y^i$ as the only inputs. As a consequence, producer prices are fixed; we normalize them to one. Given this simple structure, any demanded quantities for goods $c$ and $x$ are produced, and we can neglect the production side in what follows.

**Preferences:** Individuals derive utility from the *absolute* consumption of $c$ and $x$. In addition, they care about *relative* consumption $\Delta$, i.e., about how their own consumption levels compare to that of others. As suggested by recent empirical evidence, the importance of consumption comparisons may differ across goods (Solnick and Hemenway, 2005; Charles et al. 2009; Heffetz, 2011). We assume that good $c$ is positional, meaning that both absolute and relative consumption levels matter. Good $x$ is non-positional, and individuals only care about it's absolute consumption. Formally, preferences are represented by a utility function $U : \mathbb{R}^3 \to \mathbb{R}$, with

$$U(c, x, \Delta) = u(c, x) + \Delta. \tag{3.1}$$

In (3.1), $u(c, x)$ is absolute consumption utility, where $u : \mathbb{R}^2_+ \to \mathbb{R}$ is strictly increasing in both arguments, twice continuously differentiable, and strongly quasi-concave.[6] Denote individual consumption levels of both goods by $c^i$ and $x^i$. We define individual $i$'s relative consumption $\Delta^i$ as the difference between her own consumption of good $c$ and some reference level $r^i$:[7]

$$\Delta^i := c^i - r^i. \tag{3.2}$$

An individual's reference level is (weakly) increasing in both the average positional good consumption of her own and that of the respective other income group. As individuals with the same gross income are identical, in equilibrium, $c^i$ will coincide with average consumption of group $i$. Thus, with a slight abuse of notation, we define $r^i$ as a non-decreasing, twice continuously differentiable function $h^i : \mathbb{R}^2_+ \to K^i \subset \mathbb{R}^1$, with

$$r^i := h^i(c^i, c^j), \quad \frac{\partial h^i}{\partial c^i} \geq 0 \quad \text{and} \quad \frac{\partial h^i}{\partial c^j} \geq 0 \quad \text{for} \quad i \neq j. \tag{3.3}$$

We restrict the range of $h^i$ to $K^i := \{r^i \mid 0 \leq r^i \leq y^1 + y^2\}$, which means that reference levels cannot be larger than the economy's aggregate resource endowment and ensures the existence of an equilibrium (see Section 3.4). In addition, for at least one income type, either of the derivatives $\partial h^i/\partial c^i$ or $\partial h^i/\partial c^j$ must be strictly greater than zero for all $(c^i, c^j)$ – otherwise, all reference levels would be exogenous and a market inefficiency would not occur (see below).
Substituting (3.2) and (3.3) into (3.1), an individual's utility is

$$U^i_*(c^i, x^i, c^j) := u(c^i, x^i) + c^i - h^i(c^i, c^j) \quad \text{for} \quad i \neq j, \tag{3.4}$$

where $U^i_* : \mathbb{R}^3_+ \to \mathbb{R}$. We assume that $U^i_*$ is strongly quasi-concave and that relative consumption concerns are not "too strong" in the sense that $U^i_*$ strictly increases in $c^i$.[8] According to (3.4), individuals' utilities may – through the reference functions $h^i$ – decrease in the consumption levels of others.

[6]Strong quasi-concavity is less general than strict quasi-concavity (see, e.g., Barten and Böhm, 1982; Bilancini and Boncinelli, 2010). It ensures that the bordered Hessian matrix of $U$ is negative definite and allows us to analyze comparative statics of individual demands.

[7]We use this specific form of preferences to illustrate our main points as simple as possible. In Section 3.4, we study more general preferences.

[8]An equivalent assumption is made in Dupor and Liu (2003) and Dufwenberg et al. (2011).

**Social comparisons:** The general formulation of $h^i$ includes comparisons with the average consumption of the entire economy $\bar{c} := (c^1 + c^2)/2$ as a special case. A key feature of our formulation is, however, to allow different individuals to have different reference levels, which captures ideas from social psychology that the notion of what constitutes a "referent other" may considerably vary across individuals and social groups (Suls and Wills, 1991). According to the similarity hypothesis, for example, social comparisons are local: individuals are more concerned about the consumption levels of their immediate associates than of people who are socially distant (see, e.g., Festinger, 1954; Runciman, 1966; Frank, 1984). In this case, an individual feels envious when another person has a bigger car – but more so, if the other person is one's neighbor, colleague, friend, or family member. In our framework with two social groups, we can represent such local comparisons by letting individuals' reference levels be exclusively sensitive to the consumption of members of their own income group, i.e., $\partial h^1/\partial c^1 > 0$, $\partial h^2/\partial c^2 > 0$, and $\partial h^1/\partial c^2 = \partial h^2/\partial c^1 = 0$ (*pure within-group comparisons*).

In contrast to comparisons within similar groups, Duesenberry (1949) suggests that people are upward looking and mainly care about the consumption of individuals who are ranked socially higher. Taken to the extreme, *pure upward comparisons* would entail $\partial h^1/\partial c^2 > 0$, $\partial h^1/\partial c^1 = \partial h^2/\partial c^2 = \partial h^2/\partial c^1 = 0$.

The opposite polar case is when individuals relate only to those below them in the income ranking. As is advocated by self-enhancement theory, individuals compare with others in order to make themselves feel better, and therefore tend to compare downward as a means to increase self-esteem (Wood and Taylor, 1991). In the case of *pure downward comparisons*, we would have $\partial h^2/\partial c^1 > 0$ and $\partial h^2/\partial c^2 = \partial h^1/\partial c^1 = \partial h^1/\partial c^2 = 0$.[9] Such a reference specification also entails elements of Veblen's (1899) snobbism, where the rich's primary social motive is to behaviorally separate from the poor. While we do not take any position on these views, our modeling enables us to explicitly distinguish between different kinds of social comparisons.

[9]Strictly speaking, this reasoning considers reference levels as an active choice variable. Endogenizing reference levels (as well as the dimensions over which individuals compare) is beyond the scope of this paper and left for future research.

### 3.2.2 Behavior and equilibrium

When making consumption choices, each individual treats her reference level as exogenous.[10] Utility for a given reference level is represented by the function $\tilde{U} : \mathbb{R}^3_+ \to \mathbb{R}$, with

$$\tilde{U}(c^i, x^i, r^i) := u(c^i, x^i) + c^i - r^i. \tag{3.5}$$

We require $\tilde{U}$ to be strongly quasi-concave in $x^i$ and $c^i$, i.e., given a reference level, preferences over the positional and non-positional good are strictly convex. In addition, let $\tilde{U}\left(c^i, x^i, r^i\right) > \tilde{U}\left(0, \hat{x}^i, r^i\right)$ and $\tilde{U}\left(c^i, x^i, r^i\right) > \tilde{U}\left(\hat{c}^i, 0, r^i\right)$ for all $r^i$, $c^i > 0, x^i > 0$, $\hat{c}^i \geq 0$ and $\hat{x}^i \geq 0$, which ensures strictly positive demands for both goods for positive incomes.

Denote the set of consumption bundles available to individual $i$ by $B^i \subset \mathbb{R}^2_+$. In all policy scenarios we consider, this budget set will be compact and convex.[11] Each individual then solves

$$\max_{c^i, x^i} \tilde{U}(c^i, x^i, r^i) \quad \text{s.t.} \quad (c^i, x^i) \in B^i. \tag{3.6}$$

Since $\tilde{U}$ is strongly quasi-concave, problem (3.6) has a unique solution, defining type $i$'s demands for goods $c$ and $x$. By additive separability of $\tilde{U}$, demands do not depend on the reference level $r^i$; consumption comparisons only affect utility, but have no behavioral effects. In the following, we will repeatedly use individuals' "ordinary" or "unconstrained" demand functions $c_d\left(I\right)$ and $x_d\left(I\right)$ that solve problem (3.6) for the budget set $B_u := \{(c, x) : c + x \leq I\}$, where $I$ denotes an exogenous disposable income. We assume that both goods are normal, i.e., $\partial c_d(I)/\partial I > 0$ and $\partial x_d(I)/\partial I > 0$.

We define an equilibrium as follows:

**Definition 3.1** *An allocation $C := (c^1, x^1, c^2, x^2)$ and a corresponding pair of reference levels $(r^1, r^2)$ constitute an equilibrium if*

*(i) for every $i$, $(c^i, x^i)$ solves (3.6) contingent on $B^i$,*

[10]This assumption is standard in the literature on relative consumption concerns and analogue to price-taking behavior of atomistic individuals. Intuitively, as the number of individuals is large, they regard their own contribution to the reference level as negligible. Since reference levels are a function of the consumption of others, we could equivalently say that individuals take others' consumption levels as given.

[11]The general formulation of $B^i$ simplifies the exposition as it allows us to express the definition of an economic equilibrium as general as possible and includes each policy scenario as a special case.

*(ii)* $\sum_{i=1}^{2}\left(c^i + x^i - y^i\right) = 0$,

*(iii) for every* $i \neq j$, $r^i = h^i\left(c^i, c^j\right)$.

According to items (i) and (ii), an equilibrium allocation must maximize individuals' utility given their budgets and satisfy the economy's resource constraint. Item (iii) requires that reference levels are consistent with actual behavior (or, alternatively, that individuals foresee others' behavior correctly). With preferences as in (3.1) and (3.2), a unique equilibrium always exists.[12] When preferences are more general and reference levels do affect behavior, multiple equilibria can emerge. We will study this case in Section 3.4.

### 3.2.3 Inefficiency of the laissez-faire

When people care for relative consumption, the allocation $C_{LF} := (c_{LF}^1, x_{LF}^1, c_{LF}^2, x_{LF}^2)$ in a laissez-faire without government intervention is inefficient. To see this, first consider the set of Pareto efficient allocations $\mathcal{P}$. As shown in Appendix 3.A.1, any efficient allocation $C_* := (c_*^1, x_*^1, c_*^2, x_*^2)$ must satisfy

$$MRS(c^i, x^i) - \Gamma^i = 1 \quad \text{for} \quad i = 1, 2, \tag{3.7}$$

where

$$MRS(c^i, x^i) := \frac{u_c^i + 1}{u_x^i} \quad \text{and} \quad \Gamma^i := \left[\frac{1}{u_x^i}\frac{\partial h^i}{\partial c^i} + \frac{1}{u_x^j}\frac{\partial h^j}{\partial c^i}\right] \geq 0 \quad \text{for} \quad i \neq j.^{13} \tag{3.8}$$

We restrict $\mathcal{P}$ to allocations where both income types receive positive amounts of both goods and $c_*^1 < c_*^2$ and $x_*^1 < x_*^2$.[14] In (3.7), the left-hand side is the economy's aggregate marginal willingness to pay for an increase in $c^i$ measured in units of the non-positional good. The term $MRS\left(c^i, x^i\right)$ represents individual $i$'s "private" marginal rate of substitution between goods $c$ and $x$ for a constant reference level. The second term, $\Gamma^i$, reflects that an individual's consumption of the positional good may negatively affect others' well-being through the reference functions $h^i$, and measures the willingness to pay of others to decrease $c^i$. In the following, we will refer to $\Gamma^i$ as the marginal "social

[12] It can be shown that item (ii) is satisfied for all policies that balance the government budget.

[13] In the following, we abbreviate $u_c^i := \partial u(c^i, x^i)/\partial c$ and $u_x^i := \partial u(c^i, x^i)/\partial x$.

[14] We make the second assumption so that our distinction of the two types by their gross incomes remains meaningful. However, the assumption only serves to simplify the exposition and is not essential for any of our results.

harm" of type $i$'s positional good consumption. The right hand side of (3.7) gives the marginal social cost of $c^i$, which is constant and equal to one since we assumed a linear production technology.

In the laissez-faire, individual budget sets are $B^i_{LF} := \{(c^i, x^i) : c^i + x^i \leq y^i\}$. For an individual of type $i$, the solution to the maximization problem (3.6) is therefore given by $c^i_{LF} := c_d\,(y^i)$ and $x^i_{LF} := x_d\,(y^i)$. Using the first-order conditions for problem (3.6), it can be shown that the laissez-faire allocation satisfies

$$MRS(c^i, x^i) = 1 \quad \text{for} \quad i = 1, 2. \tag{3.9}$$

As we assume endogenous reference levels, $\Gamma^i$ is greater than zero for at least one type. As a consequence, condition (3.9) does not coincide with (3.7), and $C_{LF}$ is inefficient. Intuitively, the marginal social harms are not reflected in market prices and therefore neglected in private optimization; there is a divergence between the private and social evaluations of an individual's positional spending. Thus, whenever $\Gamma^i > 0$, income type $i$'s consumption of the positional good imposes a negative externality on others.

In addition, all individuals for whom $\Gamma^i > 0$ over-consume [under-consume] the positional [non-positional good] in the laissez-faire: by (3.9), the aggregate willingness to pay for $c^i$ at $C_{LF}$ equals $1 - \Gamma^i$, and thus falls short of the social cost. Since the utility function $U^i_*$ is strongly quasi-concave, there always exists a consumption bundle containing a slightly lower [higher] level of $c^i$ [$x^i$] that is Pareto-superior to $(c^i_{LF}, x^i_{LF})$. The inefficiency of the laissez-faire generates scope for government intervention. In the following, we show that public provision of private goods can correct the external effects from individuals' positional good consumption.

## 3.3 The efficiency role of public provision

Many government-provided goods are rather less important for social comparisons. For example, health care, basic education, and old-age consumption are relatively little effective in gaining high social status compared to smart phones, clothes, and cars (see Alpizar et al., 2005; Solnick and Hemenway, 2005; Charles et al. 2009; Heffetz, 2011). We now lay out the basic argument for why publicly providing such non-positional goods can achieve Pareto improvements over the laissez-faire.

The government may provide a uniform level $g$ of the non-positional good, which is offered to all individuals free of charge and can be topped up by additional purchases on a market. However, individuals cannot resell the publicly provided level, i.e., the sum of public provision and private purchases must exceed or equal $g$.[15] To finance public provision, (lump-sum) income taxes $T^i = T(y^i)$ can be levied. Type $i$'s net income $b^i$ is then equal to $y^i - T^i$. It will be convenient to use $b^i$ as a government choice variable, rather than $T^i$ itself. We call $P = (b^1, b^2, g) \in \mathbb{R}^3_+$ a policy, which is a scheme assigning the uniform provision level and a net income to each individual. Policies must be feasible and balance the government budget:

$$G := y^1 - b^1 + y^2 - b^2 - 2g = 0. \tag{3.10}$$

We restrict the set of available policies to those where $b^2 > b^1$, so our initial notion of rich and poor remains meaningful. We further require $b^i > 0$ for $i = 1, 2$, i.e., income taxation is not exhaustive.

## 3.3.1 Individual behavior under public provision

For a given policy, individuals decide how to allocate their net income on market purchases of the two private goods. Formally, each individual solves (3.6) with the budget set now given by

$$B^i_g := \left\{(c^i, x^i) : c^i + x^i - g \leq b^i, x^i \geq g, \right\}.$$

The unique solution to this problem must satisfy

$$MRS(c^i, x^i) - 1 \geq 0 \quad \text{and} \quad \left(x^i - g\right)\left[MRS(c^i, x^i) - 1\right] = 0. \tag{3.11}$$

[15]Otherwise, public provision would be equivalent to a cash transfer, and thus, redundant in our framework.

The demand functions obtained from problem (3.6) with budget $B_g^i$ have a piecewise form and can be expressed as follows:[16]

$$c^i(b^i, g) = \begin{cases} c_d\left(b^i + g\right) & \text{if } g < x_d\left(b^i + g\right), \\ b^i & \text{if } g \geq x_d\left(b^i + g\right), \end{cases} \tag{3.12}$$

$$x^i(b^i, g) = \begin{cases} x_d\left(b^i + g\right) & \text{if } g < x_d\left(b^i + g\right), \\ g & \text{if } g \geq x_d\left(b^i + g\right). \end{cases} \tag{3.13}$$

Demand functions are continuous, but only one-sided differentiable at policies where $g = x_d\left(b^i + g\right)$, which will suffice for our proofs below. In (3.12) and (3.13), $x_d\left(b^i + g\right)$ gives the amount of the non-positional good an individual with income $b^i$ would buy if it received the value of $g$ in cash. If $g < x_d\left(b^i + g\right)$, public provision is equivalent to a cash transfer: the individual takes the publicly provided level and purchases additional units until $x^i\left(b^i, g\right) = x_d\left(b^i + g\right)$; compared to a situation with no public provision, she simply substitutes private purchases with public provision, but still buys additional units on the market as the overall demand for good $x$ has increased. If $g \geq x_d\left(b^i + g\right)$, however, the individual would by less than $g$ when given the publicly provided amount in cash. As resales are not feasible, she then spends her entire net income on the positional good. We say that an individual is constrained in her consumption choice or that private purchases of the non-positional good are "crowded out". By the normality of good $x$, the poor are crowded out at lower levels of public provision than the rich. Let

$$V^i(b^i, b^j, g) := u^i(c^i(b^i, g), x^i(b^i, g)) + c^i(b^i, g) - h^i(c^i(b^i, g), c^j(b^j, g)) \tag{3.14}$$

be the equilibrium indirect utility of type $i$. Generally, from the definition of $h^i$, $V^i$ also depends on the net income of the respective other type. Indirect utilities are continuous, but only one-sided differentiable at points where $g = x_d\left(b^i + g\right)$.

### 3.3.2 Conditions for public provision to be Pareto-improving

Our first proposition establishes that, under relatively mild assumptions on the reference functions $h^i$, public provision of the non-positional good can *always* achieve Pareto improvements over the laissez-faire:

[16] The formal derivation is equivalent to Epple and Romano (1996b) and therefore omitted.

**Proposition 3.1** *If $\partial h^1/\partial c^1 > 0$ or $\partial h^2/\partial c^1 > 0$ for all $c^1$ and $c^2$, there exists a policy $P = (b^1, b^2, g)$ with $g > 0$ that achieves a Pareto improvement over the laissez-faire.*

The intuition for Proposition 3.1 is as follows. Consider a policy scheme where any public provision level is financed by an equal-sized reduction in both types' net incomes, $b^i = y^i - g$.[17] Under this scheme, an individual is crowded out by public provision if $g \geq x_d(b^i + g) = x_d(y^i) = x^i_{LF}$. By demand functions (3.12) and (3.13), setting the public provision level equal to $g = x^1_{LF}$ then leads to the same allocation – and hence, indirect utilities – as in the laissez faire; the poor are "just" constrained by public provision and spend their entire net income on the positional good. As both goods are normal, public provision is cash-equivalent for the rich.

Now, consider a marginal increase in the provision level slightly above $x^1_{LF}$. This policy change has no effect on both types' utilities if reference levels were constant: by the continuity of individual demand functions, the rich still consume the same consumption bundle as in the laissez-faire; the poor are forced to consume slightly more of the non-positional good and less of the positional good – which has a negative effect on $u(c^i, x^i) + c^i - r^i$. But as individuals consume optimally at $g = x^1_{LF}$, this effect is of second-order and therefore negligible. However, if $\partial h^1/\partial c^1 > 0$ or $\partial h^2/\partial c^1 > 0$, the forced reduction in $c^1$ has a positive first-order welfare effect as it lowers the reference level of at least one income group. Since the proposed policy changes are always feasible, this is sufficient for the existence of a Pareto superior policy.

The condition identified in Proposition 3.1 is satisfied if the poor compare themselves with members of their own group or the rich are looking downward. Recent empirical evidence suggests that actual relative consumption preferences indeed entail elements of within-group or downward comparisons (see, e.g., Falk and Knell, 2004; Corrazini et al., 2012; Roth, 2014).

However, public provision can be valuable even if the sufficient condition is violated. To see this, let the provision scheme again be given by $b^i = y^i - g$ and suppose that the poor are purely upward-looking and the rich are entirely in-group oriented. Then, providing the non-positional good in the interval $g \in (x^1_{LF}, x^2_{LF}]$ can only have negative efficiency effects; public provision distorts the consumption choice of the poor but has no effects on people's reference levels. But if the provision level is set slightly above $x^2_{LF}$, it forces the rich to reduce their positional consumption – which *ceteris paribus*

[17] With this simple provision scheme, public provision effectively acts as a minimum consumption constraint for the non-positional good.

has positive welfare effects since $\partial h^1/\partial c^2 > 0$ and $\partial h^2/\partial c^2 > 0$. If this effect is strong enough, a Pareto improvement may occur for some $g > x^2_{LF}$. In the following example, we demonstrate that this can indeed happen for Cobb-Douglas absolute consumption utility:

**Example 1:** Assume that the sub-utility function is $u(c^i, x^i) = c^i x^i$. Individuals then maximize $\tilde{U}(c^i, x^i, r^i) = c^i x^i + c^i - r^i$. Further assume (pure) upward comparisons of the poor and (pure) within-group comparisons of the rich, i.e., $h^1(c^1, c^2) = h^2(c^2, c^1) = c^2$. Set the parameters of the model to $y^1 = 10$, and $y^2 = 10.4$. In this case, the laissez-faire allocation is $C_{LF} = (c^1_{LF}, x^1_{LF}, c^2_{LF}, x^2_{LF}) = (5.5, 4.5, 5.75, 4.75)$. Now, introducing public provision with $g = 5$ and $T^1 = T^2 = 5$ yields $(c^1, x^1, c^2, x^2) = (5, 5, 5.4, 5)$ as the equilibrium consumption allocation. The resulting utility differential is $V^1(5, 5.4, 5) - V^1(10, 10.4, 0) = 0.05 > 0$ and $V^2(5, 5.4, 5) - V^2(10, 10.4, 0) = 0.21 > 0$, i.e., public provision makes both types strictly better off. ■

Several remarks are in order. First, we considered a simple provision system where all individuals receive a uniform amount of the non-positional good. Since the poor are crowded out first by public provision, the government can correct the externality imposed by their positional spending without causing any negative efficiency effects for the rich. As Example 1 shows, this does not hold vice versa; when the public provision level is uniform, reducing the positional spending of the rich necessarily distorts the consumption choices of the poor. But if we allowed for income-dependent provision levels, the case for public provision would be strengthened. In fact, the government could then separately target each individual's social harm and always achieve Pareto improvements.

Second, we abstracted from any redistributive motives and our benchmark was the laissez-faire. However, under the conditions stated in Proposition 3.1, public provision is valuable for any initial distribution of the economy's resources on the two income types: by the same arguments as in Section 3.2.3, individuals under-consume the non-positional good for any given distribution of net incomes. As a consequence, public provision can correct this under-consumption even if we allowed for redistribution. Finally, as both goods are normal, Proposition 3.1 easily generalizes to a model with more than two income groups.

## 3.4 Peer effects in consumption

In our basic model, social comparisons have well-being effects alone: reference levels enter utility without having any effect on consumption behavior. However, findings from the empirical literature on conspicuous consumption suggest that an individual's relative position also affects her marginal propensity to consume ( Heffetz, 2011; Roth, 2014). In this section, we therefore allow for peer effects in consumption.

### 3.4.1 Preferences and behavior

Let preferences of individual $i$ be represented by a strictly increasing, twice continuously differentiable function $U : \mathbb{R}^3 \to \mathbb{R}$, where

$$U^i := U(c^i, x^i, \Delta^i). \tag{3.15}$$

Compared to Section 3.2, preferences need no longer be separable in $\Delta^i$. In addition, an individual's relative consumption is now given by the general function $\Delta : \mathbb{R}^2_+ \to \mathbb{R}$, with

$$\Delta^i := \Delta(c^i, r^i), \quad \frac{\partial \Delta}{\partial c} > 0, \quad \frac{\partial \Delta}{\partial r} < 0. \tag{3.16}$$

I.e., each individual's (utility from) relative consumption is increasing in own consumption and decreasing in the reference level. In addition to difference-comparisons used so far, the general formulation of $\Delta$ also allows for ratio-comparisons $\Delta(c, r) = c/r$ that have been frequently studied in the literature on relative consumption (see, e.g., Clark and Oswald, 1998). Reference functions $h^i$ have the same properties as in Section 3.2. Substituting (3.16) into (3.15) yields utility for constant reference levels

$$\tilde{U}(c^i, x^i, r^i) := U(c^i, x^i, \Delta(c^i, r^i)). \tag{3.17}$$

The utility function $\tilde{U}$ is assumed to satisfy the same assumptions as in Section 3.2.2. Hence, for a given policy and reference level, there exist unique and strictly positive demands for both goods. To simplify the exposition, we restrict the analysis to public provision schemes where any provision level is financed by an equal-sized reduction in both types' net incomes and no further taxes (or transfers) are employed. Given $g$ and

$r^i$, individuals then maximize (3.17) subject to $(c^i, x^i) \in \tilde{B}_g$, with

$$\tilde{B}_g^i := \left\{ (c^i, x^i) : c^i + x^i \leq y^i, x^i \geq g, \right\}.$$

The demand functions obtained from this problem have the following form:[18]

$$c^i(g, r^i) = \begin{cases} c_d^i\left(y^i, r^i\right) & \text{if } g < x_d^i\left(y^i, r^i\right), \\ y^i - g & \text{if } g \geq x_d^i\left(y^i, r^i\right), \end{cases} \tag{3.18}$$

$$x^i(g, r^i) = \begin{cases} x_d^i\left(y^i, r^i\right) & \text{if } g < x_d^i\left(y^i, r^i\right), \\ g & \text{if } g \geq x_d^i\left(y^i, r^i\right). \end{cases} \tag{3.19}$$

The crucial difference to the previous section is that the reference level $r^i$ now affects demands for goods $c$ and $x$ if an individual is not crowded out by public provision $(g < x_d^i\left(y^i, r^i\right))$.[19] A change in the positional spending of others thus also impacts on individuals' consumption choices. The comparative statics of $c_d^i$ and $x_d^i$ with respect to $r^i$ are generally unclear in sign and depend on how the marginal rate of substitution between the positional and non-positional good, $MRS\left(c^i, x^i, r^i\right) := \tilde{U}_c^i / \tilde{U}_x^i$, varies with $r^i$.[20] In this section, we will consider two polar cases: one where $MRS(c^i, x^i, r^i)$ is increasing and one where it is decreasing in the reference level. In Appendix 3.A.3, we show that[21]

$$\frac{\partial MRS\left(c^i, x^i, r^i\right)}{\partial r^i} > (<)\, 0 \quad \Longleftrightarrow \quad \frac{\partial c_d^i}{\partial r^i} > (<)\, 0.$$

Following Dupor and Liu (2003), we say that preferences exhibit "keeping up with the Joneses" (KUJ) if an individual of type $i$ spends more on the positional good if others' consumption levels rise $(\partial c_d^i / \partial r^i > 0)$. In the reverse case, which we label as "running away from the Joneses" (RAJ), the individual responds to an increase in the reference level by reducing her positional good consumption $(\partial c_d^i / \partial r^i < 0)$. We assume that the rich have higher ordinary demands than the poor, i.e., $c_d^1\left(y^1, r^1\right) < c_d^2\left(y^2, r^2\right)$ and $x_d^1\left(y^1, r^1\right) < x_d^2\left(y^2, r^2\right)$ for all $r^1$ and $r^2$.[22]

[18] To simplify the exposition, we suppress the dependence of demand functions on gross incomes $y^i$.

[19] Whenever an individual is crowded out, the reference level has no effect on consumption, i.e., $\partial c^i / \partial r^i = 0$ and $\partial x^i / \partial r^i = 0$.

[20] We abbreviate $\tilde{U}_c^i := \partial \tilde{U}(c^i, x^i, r^i) / \partial c$ and $\tilde{U}_x^i := \partial \tilde{U}(c^i, x^i, r^i) / \partial x$.

[21] From individual budget constraints, we have $\partial x_d^i / \partial r^i = -\partial c_d^i / \partial r^i$.

[22] As the ordinary demands are normal, this is generally satisfied if both types have the same reference level. However, as we allow reference levels to vary across types, it may happen that the rich

### 3.4.2 Equilibria

Again, an equilibrium is defined as a consumption allocation and pair of reference levels that satisfy the conditions in Definition 3.1. When preferences are separable in $r^i$, there exists a unique equilibrium for a given policy. With non-separable preferences as in (3.15), multiple equilibria can occur. To see this, note that condition (iii) of Definition 3.1 requires consistency of reference levels and actual behavior, i.e.,

$$\begin{aligned} &r^1 - h^1(c^1(g,r^1), c^2(g,r^2)) = 0, \\ &r^2 - h^2(c^2(g,r^2), c^1(g,r^1)) = 0. \end{aligned} \tag{3.20}$$

For every $g$, system (3.20) is a fixed-point equation in $\mathbb{R}^2$. By applying Brouwer's fixed-point theorem, there exists a pair of reference levels $(\hat{r}^1, \hat{r}^2)$ that solves (3.20).[23] However, as demands for the positional good depend on $r^i$, several pairs of reference levels may satisfy (3.20). Since each reference level pair corresponds to a unique allocation, multiple equilibrium allocations can emerge. Given a public provision level, we collect the solutions to (3.20) in the set $\mathcal{E}_g$.

In the following, we make an assumption that ensures equilibria to be locally unique – in the sense that for every $(\hat{r}^1, \hat{r}^2) \in \mathcal{E}_g$, there is no other reference pair solving (3.20) sufficiently close to it (see Appendix 3.A.3). The assumption also guarantees that individuals' ordinary demands $c_d^i$ and $x_d^i$ are normal in the presence of peer effects. Specifically, we impose:

**Assumption 3.1** *For all $r^1$, $r^2$ and $g$:*

$$(i)\, A := \left(1 - \frac{\partial h^1}{\partial c^1}\frac{\partial c_d^1}{\partial r^1}\right)\left(1 - \frac{\partial h^2}{\partial c^2}\frac{\partial c_d^2}{\partial r^2}\right) - \frac{\partial h^1}{\partial c^2}\frac{\partial c_d^2}{\partial r^2}\frac{\partial h^2}{\partial c^1}\frac{\partial c_d^1}{\partial r^1} > 0.$$

$$(ii)\, 1 - \frac{\partial h^i}{\partial c^i}\frac{\partial c_d^i}{\partial r^i} > 0 \quad for \quad i = 1, 2.$$

Assumption 3.1 allows us to express any solution to (3.20) as an implicit function of the public provision level, $\hat{\mathbf{r}}(g) = (\hat{r}^1(g), \hat{r}^2(g))$. For every $g$, we can therefore write

have a lower demand for one of the two goods. For simplicity, we exclude such cases.

[23] To see this, rewrite (3.20) as $\mathbf{r} = (r^1, r^2)' = \mathbf{h}(\mathbf{r})$, where $\mathbf{h} : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $\mathbf{h}(r) := (\tilde{h}^1, \tilde{h}^2)'$ with $\tilde{h}^i(r^1, r^2) := h^i(c^i(g, r^i), c^j(g, r^j))$. Since demand functions are continuous in $r^i$, $\mathbf{h}$ is a continuous function mapping each point $(r^1, r^2)$ of the convex and compact set $K^1 \times K^2 \in \mathbb{R}^2$ into itself, which ensures the existence of a $(\hat{r}^1, \hat{r}^2)$ that solves (3.20) for any $g$.

individual $i$'s indirect utility as

$$V^i(g) = u^i(c^i(g, \hat{r}^i(g)), x^i(g, \hat{r}^i(g)), \Delta^i(c^i(g, \hat{r}^i(g)), \hat{r}^i(g))). \tag{3.21}$$

Denote the set of equilibria in the laissez-faire ($g = 0$) by $\mathcal{E}_{LF}$. By the same arguments as in Section 3.2.3, individuals over-consume the positional and under-consume the non-positional good in every $(\hat{r}^1, \hat{r}^2) \in \mathcal{E}_{LF}$. Irrespective of whether preferences satisfy KUJ or RAJ, the source of the inefficiency is not removed by the presence of social peer effects; individuals still neglect that their own consumption may have negative impacts on the well-being of others.

### 3.4.3 Public provision

We now give a sufficient condition under which the public provision of the non-positional good can achieve Pareto improvements over any $(\hat{r}^1, \hat{r}^2) \in \mathcal{E}_{LF}$. We assume that the government can select between different laissez-faire equilibria in the sense that, for a given $(\hat{r}^1, \hat{r}^2) \in \mathcal{E}_{LF}$, reference levels do not "jump" and remain at $(\hat{r}^1, \hat{r}^2)$ whenever the public provision level is set equal to $g = x^1_{LF}$. We can state

**Proposition 3.2** *For every $(\hat{r}^1, \hat{r}^2) \in \mathcal{E}_{LF}$, there exists a Pareto-improving policy with $g > 0$ if*

*(i) preferences exhibit KUJ ($\partial c^i_d / \partial r^i > 0$) and*

*(ii) $\partial h^1/\partial c^1 > 0$ or $\partial h^2/\partial c^1 > 0$ for all $c^1$ and $c^2$.*

In addition to the presence of within-group comparisons of the poor or downward comparisons of the rich, the sufficient condition identified in Proposition 3.2 requires preferences to satisfy KUJ when there are peer effects in consumption; positional consumption choices must be strategic complements: if one individual increases her status consumption, other people will follow to spend more on goods that confer a high relative standing. The logic behind this result is similar to that of Proposition 3.1. When the government sets the provision level slightly above a given laissez-faire level $x^1_{LF}$, the poor are forced to slightly reduce their positional spending – which *ceteris paribus* has positive welfare effects if $\partial h^1/\partial c^1 > 0$ or $\partial h^2/\partial c^1 > 0$. However, the reduced positional consumption of the poor may induce a behavioral change for the rich who are not constrained at $g = x^1_{LF} < x^2_{LF} = c^2_d(y^2, r^2)$. If preferences exhibit RAJ ($\partial c^i_d/\partial r^i < 0$), the rich would react to the decrease in $c^1$ by increasing their positional spending, which would call for a rise in individuals' reference levels. If this counteracting effect is strong enough, a decrease in $c^1$ might increase reference levels in equilibrium and lower indirect utilities. Under KUJ, $\partial c_d^i/\partial r^i > 0$, $c^1$ and $c^2$ tend to move in the same direction. Crowding out positional consumption of the poor would also lower that of the rich and reinforce the initial positive welfare effect.

Evidence for such positive "social interaction" effects can be found in the recent social networks literature, which reports conformity or bandwagon effects in various social contexts, including risky behavior, recreational activities, or labor supply decisions (for a survey, see Durlauf and Ioannides, 2010). More closely to our framework, Kuhn et al. (2011) provide evidence for KUJ effects in consumption. In the Dutch Postcode Lottery, neighbors of lottery winners (who received cash and a new BMW) increase spending on cars and exterior home renovation. Likewise, using data from a randomized conditional cash transfer program in Indonesia, Roth (2014) finds that the expenditure share of visible goods rises for untreated households whose reference group's visible consumption is exogenously increased. In that sense, available empirical evidence suggests that public provision is a valuable instrument even when peer effects are present.

## 3.5 Taxation of the positional good

The rationale for the public provision of non-positional goods demonstrated so far might be driven by an arbitrary restriction on available policy instruments. In fact, a standard policy recommendation to address inefficiencies from status consumption is to levy a tax on the positional good (Ireland, 1994; Hopkins and Kornienko, 2004; Micheletto, 2008; Eckerstorfer and Wendner, 2013). To underscore the efficiency-enhancing potential of public provision, we now study the case where taxation of the positional good is possible. For simplicity, we will abstract from peer effects in consumption and return to the basic model of Section 3.2. Specifically, in addition to income taxes and public provision, the government can levy a uniform per unit tax $t$ on good $c$. Each individual then faces a consumer price of $p(t) = 1 + t$. A policy is described by the vector $P = (b^1, b^2, g, t)$ and, again, policies must be feasible and balance the government budget:

$$G := y^1 - b^1 + tc^1 + y^2 - b^2 + tc^2 - 2g = 0. \tag{3.22}$$

To shorten the presentation, we specify individual behavior and demand functions in Appendix 3.A.4. Denote the indirect utilities of the rich and the poor respectively by $V^2\left(b^2, b^1, g, p(t)\right)$ and $V^1\left(b^1, b^2, g, p(t)\right)$.

To see whether there is a case for public provision when a tax on the positional good is available, we start from an initial situation where the government can only make use of consumption and income taxes and sets these instruments optimally. We then study whether allowing for positive levels of public provision can induce a Pareto improvement over such an initial situation.

In the absence of public provision ($g = 0$), a policy $P = (b^1, b^2, 0, t)$ is defined as optimal if it maximizes $V^1\left(b^1, b^2, 0, p(t)\right)$ given that $V^2\left(b^2, b^1, 0, p(t)\right)$ does not fall below a minimum level $\bar{U}^2$ and the government budget constraint holds. Solving this problem for varying minimum utility levels gives the set of optimal policies $\mathcal{S}$. We assume that there exists a unique interior solution for every $\bar{U}^2$. Denote an element in $\mathcal{S}$ by $P_0 = (b_0^1, b_0^2, 0, t_0)$ and the corresponding allocation by $C_0 = (c_0^1, x_0^1, c_0^2, x_0^2)$. We restrict $\mathcal{S}$ to policies where $b^2 > b^1$, such that our distinction of the two types in rich and poor remains meaningful.

In Appendix 3.A.4, we show that for any $P_0 \in \mathcal{S}$, the optimal tax on the positional good can be (implicitly) written as a weighted average of both groups' social harms of positional consumption:

$$t_0 = [\alpha\Gamma^1 + (1-\alpha)\Gamma^2] > 0, \tag{3.23}$$

where the weights $\alpha \in (0,1)$ and $(1-\alpha) \in (0,1)$ are defined in Appendix 3.A.4.[24]

If optimal policies $P_0$ suffice to implement Pareto efficient allocations, public provision – as well as any other policy instrument – would be redundant. As a benchmark, our next proposition identifies when this is the case:

**Proposition 3.3** *A policy $P_0 \in \mathcal{S}$ induces a Pareto efficient allocation $C_* \in \mathcal{P}$ if and only if $\Gamma^1|_{C=C_0} = \Gamma^2|_{C=C_0}$.*

When both income groups impose identical marginal social harms, a uniform tax on the positional good – combined with appropriate income taxes – is sufficient to restore efficiency: as individuals choose both goods according to $MRS(c^i, x^i) = 1 + t$, a tax of $t = \Gamma^1 = \Gamma^2$ implies $MRS(c^i, x^i) - \Gamma^i = 1$, which coincides with the efficiency condition

[24] This formula is familiar from the literature on the optimal taxation of consumption externalities (see, for example, Diamond, 1973; Balcer, 1980; Micheletto, 2008; Eckerstorfer and Wendner, 2013).

(3.7). Conversely, if $\Gamma^1 \neq \Gamma^2$, both income groups exert different marginal externalities, and uniform consumption taxes can never implement efficient allocations.[25] In such cases, public provision as an additional policy instrument can be valuable.

The question is which types of social comparisons imply identical social harms. Using the definition in (3.8), $\Gamma^1 = \Gamma^2$ is equivalent to

$$\frac{1}{u_x^1}\left(\frac{\partial h^1}{\partial c^1} - \frac{\partial h^1}{\partial c^2}\right) = \frac{1}{u_x^2}\left(\frac{\partial h^2}{\partial c^2} - \frac{\partial h^2}{\partial c^1}\right). \tag{3.24}$$

It is straightforward to check that condition (3.24) holds if every individual compares her own consumption with the economy's average $\bar{c}$. More generally, there is no role for public provision if, for both income types, the identity of the individual who purchases the positional good is irrelevant, i.e., if $\partial h^i/\partial c^i = \partial h^i/\partial c^j$ for all $i \neq j$. In this case, it would be immaterial to an individual whether it is her neighbor who buys a new car or a socially more distant member in society. However, both casual observation and recent empirical evidence question that actual social comparisons are of this type (see, e.g., Clark and Senik, 2010). In fact, most of the examples we discussed in Section 3.2 would imply different marginal social harms. This clearly holds for pure upward and downward comparisons (where one of $\Gamma^1$ or $\Gamma^2$ is equal to zero), but is generally also true for pure within-group comparisons and all intermediate types with $\partial h^i/\partial c^i \neq \partial h^i/\partial c^j$.[26] Hence, in general, there is room for public provision of private goods even if an optimal uniform tax on the positional good is in place.

The next proposition demonstrates that if the poor impose the larger social harm, public provision of the non-positional good can be always Pareto-improving compared to policies $P_0$:

**Proposition 3.4** *For every $P_0 \in \mathcal{S}$, if $\Gamma^1|_{C=C_0} > \Gamma^2|_{C=C_0}$, there exists a policy $P$ with $g > 0$ which is Pareto-superior to $P_0$.*

[25] A well-established result in optimal taxation theory states that, if taxes on an externality-generating good are allowed to vary across consumers such that every individual can be assigned a personalized price, consumption taxes can implement Pareto efficient allocations (Diamond, 1973; Sandmo, 1975; Green and Sheshinski, 1976). However, individual-specific consumption taxes are difficult to implement in practice: this would, for instance, require that every customer can be charged with a different price at the cash register or that governments can observe the identity of a purchaser (i.e., who consumes how much of a particular good), which is typically considered administratively or politically infeasible (Ireland, 1994; Micheletto, 2008; Eckerstorfer and Wendner, 2013). We therefore only study uniform consumption taxes.

[26] An externality with $\partial h^i/\partial c^i \neq \partial h^i/\partial c^j$ for at least one $i$ is called non-atmospheric. The implications of non-atmospheric externalities for the theory of optimal commodity taxation have recently been studied in, e.g., Micheletto (2008) and Eckerstorfer and Wendner (2013).

To get an intuition for Proposition 3.4, consider a policy $P_0 \in \mathcal{S}$ and the corresponding allocation $C_0$. By setting the public provision level slightly above $x_0^1$ and lowering every individual's net income by an equal-valued amount, the government can force the poor to reduce their positional spending, which has a social benefit given by $\Gamma^1$. In contrast to the case without consumption taxes, however, an additional effect emerges: the lower positional consumption of the poor creates a tax revenue loss equal to $t_0$. If the marginal benefit from crowding-out the poor outweighs the revenue loss (i.e., if $\Gamma^1 > t_0$), one can always find a feasible policy that induces a Pareto improvement over $P_0$. Using formula (3.23) for the optimal tax $t_0$, the requirement $\Gamma^1 > t_0$ is equivalent to $\Gamma^1 > \Gamma^2$ – the condition stated in Proposition 3.4.

From the definitions in (3.8), $\Gamma^1 > \Gamma^2$ is satisfied if

$$\frac{1}{u_x^1}\left(\frac{\partial h^1}{\partial c^1} + \frac{\partial h^2}{\partial c^1}\right) > \frac{1}{u_x^2}\left(\frac{\partial h^2}{\partial c^2} + \frac{\partial h^1}{\partial c^2}\right). \tag{3.25}$$

As opposed to the case where positional good consumption cannot be taxed, condition (3.25) involves information about the relative strength of $\partial h^i/\partial c^i$ and $\partial h^i/\partial c^j$. In particular, public provision is always desirable when the rich have sufficient degrees of downward orientation or the poor are sufficiently concerned with the consumption levels of members from their own group (*ceteris paribus*, $\partial h^2/\partial c^1$ or $\partial h^1/\partial c^1$ must be large enough).[27]

While we are not aware of any direct evidence providing estimates of individuals' social harms, such preferences scenarios appear to be relevant in important social contexts. For example, evidence from social psychology suggests that people have strong tendencies to refer downward in the sense of comparing their own consumption to that of those behind in the income hierarchy (see, e.g., Falk and Knell, 2004, and the references therein). Likewise, the importance of in-group comparisons is one of the basic assumptions of social identity theory, which recently found entrance into economics (Akerlof and Kranton, 2000).

However, public provision of non-positional goods might be desirable even if $\Gamma^2 > \Gamma^1$. We demonstrate this in a numerical example in Appendix 3.A.7, where we assume pure upward comparisons of the poor. As in the case without consumption taxes, public provision then needs to constrain both the rich and the poor. In sum, for a wide range of social comparison types, there exists a strong rationale for public provision even if

[27] Condition (3.25) globally holds in the polar cases of pure downward comparisons of the rich or pure within-group comparisons of the poor.

governments can implement optimal uniform taxes on the positional good.[28]

## 3.6 Implementation of Pareto efficient allocations

As shown in previous sections, public provision can be a valuable instrument to attain Pareto improvements in the presence of relative consumption concerns. We derived our results under mild assumptions on the type of the public provision system: the provision level was uniform and individuals were allowed to purchase additional units of the non-positional good on the market. In this section, we ask whether such simple provision schemes can also implement Pareto efficient allocations. Again, we abstract from peer effects in consumption and study the cases with and without a consumption tax separately. When governments have access to a tax on the positional good, we only consider scenarios where $\Gamma^1 \neq \Gamma^2$, as public provision is redundant otherwise. We can state

**Proposition 3.5** *For any $C_* \in \mathcal{P}$,*

*(i) if a tax on the positional good is not available, there exists a policy $P = (b^1, b^2, g)$ with $g > 0$ that implements $C_*$ if and only if $\Gamma^1 > 0$ and $\Gamma^2 = 0$.*

*(ii) if a tax on the positional good is available, there exists a policy $P = (b^1, b^2, g, t)$ with $g > 0$ that implements $C_*$ if and only if $\Gamma^1|_{C=C_*} > \Gamma^2|_{C=C_*}$. This policy entails $g = x^1_*$ and $t = \Gamma^2|_{C=C_*}$.*

In general, if both income groups impose different social harms, the government needs to target each type's marginal external effect $\Gamma^i$ differently. As a consequence, a single uniform policy instrument like public provision typically fails to eliminate all the inefficiencies from positional good consumption. However, as demonstrated in item (i) of Proposition 3.5, there are special circumstances where public provision alone can support efficient allocations. This happens if and only if $\Gamma^1 > 0$ and $\Gamma^2 = 0$, i.e., the poor make pure within-group comparisons or the rich are entirely downward-looking. In such cases, one needs a policy instrument that can distort the consumption choices of the poor, but leaves the rich unconstrained. With our simple public provision system, this is possible: as both goods are normal and individuals are allowed to top up the

[28] It can be shown that Propositions 3.3 and 3.4 extend to a framework with more than two income groups: if and only if $\Gamma^i$ is identical for all types, public provision is redundant; if $\Gamma^i$ (weakly) decreases as individuals get richer, public provision can always achieve Pareto improvements over policies $P_0$.

publicly provided level, by setting $g = x^1_*$, the government can force the poor to choose the intended consumption bundle $(c^1_*, x^1_*)$ without constraining the rich.

While interesting, however, $\Gamma^2 = 0$ is a strong condition. It fails whenever the poor are upward looking or the rich are within-group oriented, cases we cannot exclude a priori.[29] If we allow for a consumption tax as an additional instrument, Pareto efficient allocations can be achieved even if $\Gamma^2 > 0$. As stated in item (ii) of Proposition 3.5, this requires $\Gamma^1 > \Gamma^2$. The intuition for item (ii) is as follows. As the poor are crowded out by lower levels of public provision than the rich, any policy intended to implement a given efficient allocation must involve $g = x^1_*$ and $t = \Gamma^2$: the public provision system corrects consumption choices of the poor, while the tax corrects choices of the rich.[30] But such a policy is incentive compatible if and only if $\Gamma^1 > \Gamma^2$. In the reverse case, a consumption tax equal to $\Gamma^2$ would be "too large" for the poor; at $g = x^1_*$, they would buy additional units on the market, and no combination of consumption tax and public provision could support $C_*$.

When $\Gamma^2 > \Gamma^1$, we therefore need stronger assumptions on available provision systems. One possibility would be to allow for income-dependent public provision levels. Any efficient allocation could then be implemented by provision levels equal to $x^1_*$ and $x^2_*$. However, income-specific provision systems might be infeasible due to high administrative costs or political economy considerations. Alternatively, governments could maintain the assumption of uniform provision, but prohibit additional purchases of the government-provided good on the market. Individuals would then face a choice between accepting the publicly provided level or "opting-out" of the public system and purchasing the non-positional good entirely on the market at own expenses. Educational services are often available in such "dual" provision systems, where parents can send their children to either a public or a private school or kindergarten. With a provision system of this type, Pareto efficient allocations might be attainable even if the marginal social harm is stronger among the rich. This requires, however, that the poor [the rich] indeed stay in [out of] the public system at policies with $g = x^1_*$ and $t = \Gamma^2$ – a condition which does not hold for all efficient allocations $C_* \in \mathcal{P}$.

[29] E.g., evidence for upward social comparisons can be found in Bowles and Park (2005), Corrazini et al. (2012) or Drechsel-Grau and Schmid (2014).

[30] As we only consider efficient allocations where $x^1_* < x^2_*$, the rich are unconstrained at $g = x^1_*$ and choose $MRS(c^2, x^2) = 1 + t$. It is straightforward to check that a tax equal to $\Gamma^2$ implies the efficiency condition (3.7).

A corollary of item (ii) is that public provision would even outperform consumption taxes in scenarios where $\Gamma^1 > 0$ and $\Gamma^2 = 0$: as a tax always distorts the consumption choices of both income groups, it cannot target $\Gamma^1$ separately. We therefore have a situation where a quantity-based policy strictly dominates a price instrument; to internalize externalities from positional spending, governments should publicly provide health care or education, but not tax smart phones, cars, or jewelry.[31]

## 3.7 Public provision of the positional good

In previous sections, the publicly provided good was *non-positional*. In line with empirical evidence, this characteristic can be reasonably attributed to goods like health care, health insurance, elderly care, or old-age consumption. The case of education is, however, perhaps less clear-cut. While the majority of studies in the recent literature on conspicuous consumption classifies education as non-positional (see, e.g., Charles et al., 2009; Heffetz, 2011; Khamis et al., 2012; Friehe and Mechtel, 2014), Frank (1985a) argues that it is precisely the positional aspect of education that may justify government interventions.[32,33] In his argument, education is considered as a signal for some unobservable desirable trait such as intelligence, ability or motivation. Individuals with highest educational levels then have the best job opportunities in the labor market. Out of the fear that their children will fail to receive one of these scarce positions, parents *ceteris paribus* have an incentive to increase spending on the resources devoted their children's education. But as other parents act in the same way, an "educational rat race" would be unleashed where every child's relative position remains unchanged in equilibrium. By public provision, the government can prevent parents to spend excessively on educational services – which frees resources that can be used for other welfare-enhancing items.

In this section we formalize Frank's argument. We set out the conditions under which public provision of the *positional* good can be desirable, with and without a tax on the

[31] As a caveat, it should be stressed that our results do not generalize to a model with more than two income groups (and hence, potentially more than two marginal external effects). Intuitively, governments cannot correct all the different externalities with only two uniform policy instruments.

[32] In fact, the only exception in this literature is Roth (2014). In a study using Indonesian household survey data, education is placed second in some visibility ranking scale, and thus, considered a positional good.

[33] Generally, what makes a good positional may also depend on social and cultural environment (Heffetz, 2012).

positional good.[34]

### 3.7.1 Framework

Individuals' preferences are again as in Section 3.2. Let the uniform provision level of the positional good be denoted by $e$. Since individuals over-consume the positional good in the laissez-faire, public provision must *reduce* individuals' consumption of good $c$ in order to achieve Pareto improvements. However, distorting positional spending downwards is impossible with a top up provision system: whenever the government provides a lower level than consumed in the laissez-faire, individuals would top up $e$ to $c^i_{LF}$. We therefore consider a provision system where individuals must decide whether to accept the publicly provided level $e$ or to "opt out" of the public system and to purchase the positional good on a market; parents can send their children either to a public or private school, but not both. The government further has access to a per unit tax $t$ on good $c$ and income taxes $T^i = T(y^i)$.

The chronology of events is as follows. In a first stage, the government specifies a policy $P = (b^1, b^2, e, t)$. Given $P$, in the second stage, individuals decide whether to consume the positional good in or out of the public system (system choice). In the third stage, individuals spend their net incomes to maximize utility (consumption choice). We assume that in stages 2 and 3, individuals treat their reference levels as given.

We solve the model backwards. If an individual has decided to consume the positional good in the public system, it has $e$ units of good $c$ and – as additional purchases are not feasible – spends the entire net income on the non-positional good. For a given reference level, the individual thus obtains utility $V^i_{in} := u(e, b^i) + e - r^i$. If the individual opts out of public provision, it chooses the bundle $(c^i, x^i)$ to maximize $u(c^i, x^i) + c^i - r^i$ subject to $pc^i + x^i = b^i$. By our assumptions on preferences, there exist demand functions $c_d(b^i, p)$ and $x_d(b^i, p)$. Observe that – in contrast to a top up provision system – $e$ is not an argument of the demand functions. The individual then enjoys utility $V^i_{in} := u(c_d(b^i, p), x_d(b^i, p)) + c_d(b^i, p) - r^i$.

In the second stage, each individual decides whether to take $e$ or to buy good $c$ on the market. For a given policy, there exists a unique provision level $\hat{e}^i = \hat{e}(b^i, p)$ such that individual $i$ stays in (opts out of) the public system if $e \geq (<)\, \hat{e}^i$.[35] The critical level is

[34] Interestingly, though Frank (1985a) advocates taxes on other luxury items such as yachts and jewelry, he does not suggest this price instrument to regulate overspending in education, maybe since this involves the contra-intuitive result that education should be taxed rather than subsidized.

[35] To see this, note that at $e = 0$, $V^i_{out} > V^i_{in}$ by the assumption that both goods are essential.

implicitly determined by

$$u(\hat{e}^i, b^i) + \hat{e}^i - r^i = u(c_d(b^i, p), x_d(b^i, p)) + c_d(b^i, p) - r^i. \tag{3.26}$$

As individuals must buy the positional good at own expenses when they opt out, $\hat{e}^i < c_d(b^i, p)$. Further, $\hat{e}^i$ is increasing in net income.[36]
For a given policy, individual demand functions can thus be expressed as:

$$c^i(b^i, p, e) = \begin{cases} c_d(b^i, p) & \text{if } e < \hat{e}^i, \\ e & \text{if } e \geq \hat{e}^i, \end{cases} \tag{3.27}$$

$$x^i(b^i, p, e) = \begin{cases} x_d(b^i, p) & \text{if } e < \hat{e}^i, \\ b^i & \text{if } e \geq \hat{e}^i. \end{cases} \tag{3.28}$$

Denote the number of individuals who choose public provision by $N \in [0, 2]$. As unique critical levels $\hat{e}^i$ exist, $N$ is uniquely determined for every given policy, and we can write it as a function of policy variables, i.e., $N = N(b^1, b^2, e, t)$. We define an equilibrium of the economy as follows:

**Definition 3.2** *An allocation $C = (c^1, c^2, x^1, x^2)$, a corresponding pair of reference levels $(r^1, r^2)$ and a policy $P = (b^1, b^2, e, t)$ constitute an equilibrium if*

*(i) for every $i$, $(c^i, x^i)$ solves the individual maximization problems at stages 2 and 3 for given $P$,*

*(ii) $\sum_{i=1}^{2}(x^i + c^i - y^i) = 0$,*

*(iii) for every $i \neq j$, $r^i = h^i(c^i, c^j)$,*

*(iv) $y^1 - b^1 + tc^1 + y^2 - b^2 + tc^2 - N(b^1, b^2, e, t)p\, e = 0$.*

In equilibrium, indirect utilities are

$$\begin{aligned} \tilde{V}_a^i := & u^1\left(c^i(b^i, p(t), e), x^i(b^i, p(t), e)\right) \\ & + c^i(b^i, p(t), e) - h^i\left(c^i(b^i, p(t), e), c^j\left(b^j, p(t), e\right)\right) \quad \text{for} \quad i \neq j, \end{aligned} \tag{3.29}$$

This inequality is reversed when $e$ is sufficiently large: if $e$ is set equal to $c_d(b^i, p)$, $V_{out}^i < V_{in}^i$ as $b^i > x_d(b^i, p)$. By the continuity of $V_{out}^i$ and $V_{in}^i$, $\hat{e}^i$ exists. Since $V_{out}^i$ is independent of $e$ while $V_{in}^i$ strictly increases, $\hat{e}^i$ is unique.

[36] This is seen by implicitly differentiating (3.26) respect to $b^i$ and use the fact that, along an indifference curve, the marginal utility of good $c$ is declining when both goods are normal (for a detailed proof, see Epple and Romano, 1996a).

where $a \in \{in, out\}$ indicates whether an individual chooses public provision or opts out.

### 3.7.2 Results

Our benchmark is the laissez-faire or the optimal tax solution $P_0$, depending on whether the government can tax the positional good or not. The efficiency effects of public provision crucially depend on the behavior of high-income individuals at a given provision level. If the rich are not attracted by the public system when $e$ is set to the respective level the poor consume in the laissez-faire or at the optimal tax solution, we can derive conditions on relative consumption preferences for public provision to be Pareto-improving:

**Proposition 3.6** *Assume that $c^1_{LF} < \hat{e}(y^2, 1)$ and $c^1_0 < \hat{e}(b^2_0, p_0)$. Then,*

*i) without consumption taxes, there always exist a Pareto-improving policy with $e > 0$ if $\partial h^1/\partial c^1 > 0$ or $\partial h^2/\partial c^1 > 0$,*

*ii) if a tax on good $c$ is available, for a given $P_0 \in \mathcal{S}$, there exists a Pareto-improving policy if and only if $\Gamma^1|_{C_0} \neq \Gamma^2|_{C_0}$.*

The logic behind Proposition 3.6 is as follows. When the benchmark is the laissez-faire, the government can always reduce the positional good consumption of the poor by setting the public provision level slightly below $c^1_{LF}$ and lowering net income $b^1$ by an equal-valued amount. If the rich stay out of the public system at this provision level, their consumption choice is not affected, and a Pareto improvement is attained if $c^1$ matters for social comparisons.

A similar argument applies when governments can tax the positional good. If the public provision level is set to $c^1_0$ and combined with an equal-value reduction in $b^1$, the poor are always attracted to the public system. If the rich continue to consume on the market, a Pareto improvement over policies $P_0$ is always possible if both groups' social harms differ. In cases where $\Gamma^2 > \Gamma^1$, the optimal tax $t_0$ is "too high" for the poor in the sense that they would under-consume the positional good. Hence, by marginally increasing the provision level, the government can encourage $c^1$. In the reverse case, however, the optimal tax induces the poor to over-consume good $c$ at the optimal tax solution, and lowering their positional consumption is desirable. In contrast to Section 3.5, a reduction in $c^1$ is possible through public provision since individuals cannot top up the

level $e$ on the market; in an "opt out" provision system the government has perfect control over the consumption choices of the poor – without affecting the rich.
Whenever the rich are driven into the public system at provision levels of $c^1_{LF}$ or $c^1_0$, we cannot derive conditions for public provision to be efficiency-enhancing that depend only on relative consumption preferences: while the rich's positional good consumption will be reduced in such cases, they would at the same time be pushed away from their desired consumption bundle. To assess which of these opposing effects dominates, information on the form of individual preferences and the parameters of the model is necessary.

## 3.8 Conclusion

The public provision of specific private goods is often justified by paternalistic or merit good arguments: from the perspective of some "outside observer" – e.g., the government, an altruistic donor, or even a person's own future self – consumers spend insufficient on goods like education, health care, or old-age consumption when left to their own devices. After reviewing the available empirical evidence, Currie and Gahvari (2008) conclude that paternalism or merit goods indeed seem to be the leading explanation for public provision. However, a common criticism against paternalistic policies is that they conflict with principles of consumer sovereignty.
In this paper, we offered an alternative rationale that fully respects individual preferences. In line with recent empirical evidence, people who care for relative positions devote inefficiently high shares of their budgets to positional or status goods, while spending "too little" on other, welfare-enhancing uses (Frank and Heffetz, 2011). By publicly providing (non-positional) private goods, governments can correct such inefficient consumer choices. In contrast to paternalistic approaches, individuals would then agree with the introduction of some form of public provision system – as it constitutes a device to collectively reduce wasteful spending while maintaining relative consumption or status positions.
One might object that the inefficiencies from relative consumption are better addressed by taxing the source of the inefficiency directly, rather than applying indirect policies like public provision (Sandmo, 1975). Our results reveal, however, that the public provision of private goods can play an important role even if taxes on positional goods are available. In particular, this happens in all scenarios where the marginal social harms of positional good consumption differ across social groups, which is satisfied for

sufficiently asymmetric upward, downward, or within-group comparisons. Moreover, we identified situations where the indirect instrument of public provision may even dominate the direct taxation of the positional good.

Our paper can therefore be seen as a first step towards an analysis of the relative merits of price vs. quantity instruments in environments where people have status concerns - a question that has received considerable attention in the broader field on behavioral economics (see, e.g, Farhi and Gabaix, 2015). But in order to provide robust policy recommendations, further research is necessary. First, (more) information about the intensities of marginal social harms is needed. Whereas existing empirical evidence suggests that people are indeed upward, downward, or within-group oriented, we are not aware of any estimates of the relative strength of the different types of social comparisons. Second, a thorough comparison of different policy instruments would require a more general model. In fact, with more than two income classes or goods, an optimal combination of consumption taxes and public provision would generally be second-best in that it cannot implement Pareto efficient allocations. A characterization of such optimal second-best policies as well as an analysis of alternative policy instruments (taxes or price-subsidies on other goods) is left as an avenue for future research.

# Appendix 3.A

## 3.A.1 Pareto efficient allocations

We can identify Pareto efficient allocations $C_*$ by maximizing $U_*^1$ given that (i) $U_*^2$ does not fall below a level $\bar{U}^2$ and (ii) the economy's resource constraint holds. Varying $\bar{U}^2$ then gives the whole set $\mathcal{P}$ of Pareto efficient allocation. Formally, any $C_*$ solves

$$\max_{c^1,x^1,c^2,x^2} U_*^1(c^1,x^1,c^2) \qquad \text{s.t.} \tag{3.30}$$

$$\begin{aligned}
\text{(i)} : &\quad U^2(c^2,x^2,c^1) \geq \bar{U}^2, \\
\text{(ii)} : &\sum_{i=1}^{I}(y^i - c^i - x^i) \geq 0.
\end{aligned}$$

Since $U_*^1$ is strongly quasi-concave and the constraint set in convex, closed and bounded, there exists a unique Pareto efficient allocation for every given $\bar{U}^2$. Define $\mathcal{P}$ as the set of allocations $C_*$ such that there exists $\bar{U}^2$ and the allocation solves (3.30).
The Lagrangian for problem (3.30) is

$$\begin{aligned}
L = &\, U_*^1(c^1,x^1,c^2) + \mu\left[U_*^2(c^2,x^2,c^1) - \bar{U}^2\right] \\
&+ \lambda\sum_{i=1}^{I}(y^i - c^i - x^i),
\end{aligned} \tag{3.31}$$

where $\mu$ and $\lambda$ denote the Lagrange-multipliers associated with the utility and resource constraint, respectively. An interior solution $(c_*^1, x_*^1, c_*^2, x_*^2) \gg 0$ must satisfy the first-order conditions

$$\frac{\partial L}{\partial c^1} = u_c^1 + 1 - \frac{\partial h^1}{\partial c^1} - \mu^*\frac{\partial h^2}{\partial c^1} - \lambda^* = 0, \tag{3.32}$$

$$\frac{\partial L}{\partial x^1} = u_x^1 - \lambda^* = 0, \tag{3.33}$$

$$\frac{\partial L}{\partial c^2} = -\frac{\partial h^1}{\partial c^2} + \mu^*\left(u_c^2 + 1 - \frac{\partial h^2}{\partial c^2}\right) - \lambda^* = 0, \tag{3.34}$$

$$\frac{\partial L}{\partial x^2} = \mu^* u_x^2 - \lambda^* = 0. \tag{3.35}$$

From (3.33) and (3.35), we have $\lambda^* = u_x^1$ and $\mu^* = \lambda^*/u_x^2$. Plugging these expressions into (3.32) and (3.34) gives condition (3.7).

### 3.A.2 Proof of Proposition 3.1

We have to prove that under the conditions stated in Proposition 3.1, there always exists a policy $P = (b^1, b^2, g)$ with $g > 0$ that achieves a Pareto improvement over the laissez-faire with $P_{LF} = (y^1, y^2, 0)$. Consider the policy scheme where $b^i = y^i - g$ for $i = 1, 2$. We will show that under this scheme, setting the public provision level $g$ slightly above $x^1_{LF}$ raises the utility of at least one type compared to $P_{LF}$ if $\partial h^1/\partial c^1 > 0$ or $\partial h^2/\partial c^1 > 0$.

To see this, note that with $b^i = y^i - g$, we have $x_d(b^i + g) = x_d(y^i) = x^i_{LF}$. Hence, an individual of type $i$ is crowded out by public provision if $g \geq x^i_{LF}$. Consider a change from $P_{LF}$ to policy $P_r = (b^1_r, b^2_r, g_r) = (y^1 - x^1_{LF}, y^2 - x^1_{LF}, x^1_{LF})$. At $P_r$, both types choose the same consumption bundle as in the laissez-faire: since $g_r = x^1_{LF}$, by demand functions (3.12) and (3.13), individuals of type 1 are just crowded out by public provision and choose $x^1\left(b^1_r, g_r\right) = g_r = x^1_{LF}$ and $c^1\left(b^1_r, g_r\right) = b^1_r = y^1 - x^1_{LF} = c^1_{LF}$. As $y^2 > y^1$, normality of good $x$ implies $g_r < x_d\left(y^2\right)$. Thus, demanded quantities of type 2 individuals are $c^2\left(b^2_r, g_r\right) = c_d\left(y^2\right) = c^2_{LF}$ and $x^2\left(b^2_r, g_r\right) = x_d\left(y^2\right) = x^2_{LF}$. As a consequence, at $P_r$ each type's utility is the same as in the laissez-faire. Substitution of policy $P_r$ into the government budget constraint yields $y^1 - c^1_{LF} - x^1_{LF} + y^2 - c^2_{LF} - x^2_{LF} = 0$. Therefore, $P_r$ is feasible.

Now, consider a change from $P_r$ to policy $P_p = (b^1_p, b^2_p, g_p) = (b^1_r + db^1, b^2_r + db^2, g_r + dg)$, where $dg > 0$ and $dg \to 0$. As $db^i = -dg$ for both types, policy $P_p$ is feasible. Since $g_p > x^1_{LF}$, under the one-to-one policy scheme, individuals of type 1 remain crowded out after the policy change. By the continuity of demand functions (3.12) and (3.13), individuals of type 2 are not crowded out by public provision at $P_p$. Hence, at the two policies $P_r$ and $P_p$, both types' demands are, respectively, given by $c^1(b^1, g) = b^1$, $x^1(b^1, g) = g$, $c^2(b^2, g) = x_d(b^2 + g)$ and $x^2(b^2, g) = x_d(b^2 + g)$. Inserting these demands into (3.14) gives indirect utilities

$$V^1(b^1, b^2, g) = u(b^1, g) + b^1 - h^1(b^1, c_d(b^2 + g)), \tag{3.36}$$

$$V^2(b^2, b^1, g) = u(c_d(b^2 + g), x_d(b^2 + g)) + c_d(b^2 + g) - h^2(c_d(b^2 + g), b^1). \tag{3.37}$$

Using $b^i = y^i - g$, the change in indirect utilities induced by the switch from policy $P_r$

to $P_p$ can be represented by differentiating (3.36) and (3.37) with respect to $g$ at $P_r$:

$$\frac{dV^1}{dg} = -(u_c^1 + 1) + x_x^1 + \frac{\partial h^1}{\partial c^1}, \tag{3.38}$$

$$\frac{dV^2}{dg} = \frac{\partial h^2}{\partial c^1}. \tag{3.39}$$

Since $-(u_c^i + 1) + u_x^i = 0$ at $P_r$ (see the first-order condition (3.11)) and $\partial h^1/\partial c^1 \geq 0$ and $\partial h^2/\partial c^1 \geq 0$, $dV^1/dg \geq 0$ and $dV^2/dg \geq 0$. Hence, no income type is worse off when $g$ is raised slightly above $x_{LF}^1$. If $\partial h^1/\partial c^1 > 0$ or $\partial h^2/\partial c^1 > 0$, $dV^i/dg > 0$ for at least one type, and $P_p$ achieves a Pareto improvement over $P_{LF}$. This proves the proposition. ■

### 3.A.3 Proofs for Section 3.4

#### 3.A.3.1 Derivations for KUJ and RAJ

We will show that the sign of $\partial MRS(c^i, x^i, r^i)/\partial r^i$ is equivalent to the sign of $\partial c_d^i/\partial r^i$ if an individual is not constrained by public provision, i.e., if $x^i > g$. To see this, consider individuals' maximization problem for a given $g$:

$$\max_{c^i, x^i} \quad \tilde{U}(c^i, x^i, r^i) \quad \text{s.t.} \quad (c^i, x^i) \in \tilde{B}_g^i. \tag{3.40}$$

A solution to (3.40) with $x^i > g$ must satisfy

$$-\tilde{U}_c^i + \tilde{U}_x^i = 0 \quad \text{and} \quad c^i + x^i = y^i. \tag{3.41}$$

From (3.41), we can obtain

$$\frac{\partial c^i}{\partial r^i} = \frac{\tilde{U}_{cr}^i - \tilde{U}_{xr}^i}{-\tilde{U}_{cc}^i + 2\tilde{U}_{cx}^i - \tilde{U}_{xx}^i}, \tag{3.42}$$

where we abbreviated $\tilde{U}_{cc}^i := \partial^2 \tilde{U}(c^i, y^i - c^i, r^i)/\partial c^2$, $\tilde{U}_{cx}^i := \partial^2 \tilde{U}(c^i, y^i - c^i, r^i)/\partial c \partial x$, $\tilde{U}_{xx}^i := \partial^2 \tilde{U}(c^i, y^i - c^i, r^i)/\partial x^2$, $\tilde{U}_{cr}^i := \partial^2 \tilde{U}(c^i, y^i - c^i, r^i)/\partial c \partial r$, and $\tilde{U}_{xr}^i := \partial^2 \tilde{U}(c^i, y^i - c^i, r^i)/\partial x \partial r$. Since $\tilde{U}$ is strongly quasi-concave, the denominator in (3.42) is positive (Barten and Böhm, 1982). The sign of $\partial c^i/\partial r^i$ is therefore determined by the sign of $\tilde{U}_{cr}^i - \tilde{U}_{xr}^i$. It remains to show that $\partial MRS^i(c^i, x^i, r^i)/\partial r^i > (<)\, 0 \iff \tilde{U}_{cr}^i - \tilde{U}_{xr}^i > (<)\, 0$. To see this, differentiate $MRS^i(c^i, x^i, r^i) = \tilde{U}_c^i/\tilde{U}_x^i$ with respect to

$r^i$:

$$\frac{\partial MRS^i}{\partial r^i} = \frac{\tilde{U}^i_{cr}\tilde{U}^i_x - \tilde{U}^i_c\tilde{U}^i_{xr}}{(\tilde{U}^i_x)^2} = \frac{1}{\tilde{U}^i_x}\left[\tilde{U}^i_{cr} - \frac{\tilde{U}^i_c}{\tilde{U}^i_x}\tilde{U}^i_{xr}\right]. \tag{3.43}$$

Since, $\tilde{U}^i_c/\tilde{U}^i_x = 1$ at a solution to (3.40), $\partial MRS^i\left(c^i, x^i, r^i\right)/\partial r^i > (<)\, 0 \iff \partial c^i/\partial r^i > (<)\, 0$.

#### 3.A.3.2 Proof of local uniqueness

To prove that Assumption 3.1 implies local uniqueness of equilibria, rewrite (3.20) as $F(\mathbf{r}, g) = 0$, where $F : \mathbb{R}^2 \times \mathbb{R}^1 \to \mathbb{R}^2$, $(\mathbf{r}, g) \mapsto F(\mathbf{r}, g) = (F_1(\mathbf{r}, g), F_2(\mathbf{r}, g))$ with $F_i(\mathbf{r}, g) = r^i - h^i(c^i(g, r^i), c^j(g, r^j))$. By the continuity of demand functions and $h^i$, $F$ is continuous. However, as demand functions are not differentiable at points where $g = x^i_d(y^i, r^i)$ for one or both types, the standard implicit function theorem does not apply. However, local uniqueness can be ensured by applying an implicit function theorem for non-differentiable mappings by Kumagai (1980).

To see this, let $(\hat{r}^1, \hat{r}^2, \hat{g})$ be a point such that $F(\hat{r}^1, \hat{r}^2, \hat{g}) = 0$. If there exist open neighborhoods of $(\hat{r}^1, \hat{r}^2)$ and $\hat{g}$ on which $F(\cdot, g)$ is locally one-to-one, the theorem ensures that we can express references levels as an implicit function of $g$. When no individual is crowded-out at $(\hat{r}^1, \hat{r}^2, \hat{g})$ (which includes the laissez-faire with $g = 0$), $F(\cdot, g)$ is locally one-to-one: by Assumption 3.1, the determinant of the Jacobian of $F$ with respect to $\mathbf{r}$, given by $A$, is non-zero such that $F$ is invertible. Now consider a solution to $F(\mathbf{r}, g) = 0$ where one type $i$ is just crowded out ($g = x^i_d(y^i, r^i)$). If we go into an $\epsilon$-environment of $\hat{g}$ and keep $\mathbf{r}$ fixed at $(\hat{r}^1, \hat{r}^2)$, type $i$ will either be strictly crowded-out ($g > x_d(y^i, r^i)$), or she will top up ($g < x^i_d(y^i, r^i)$). This will remain true for a sufficiently close ball around $(\hat{r}^1, \hat{r}^2)$. Since in either these cases, the determinant of the Jacobian of $F$ with respect to $\mathbf{r}$ exists and has the same sign, we can conclude that $F$ is also one-to-one around a solution where $F(\mathbf{r}, g) = 0$ is non-differentiable. Hence, Kumagai's theorem applies, and equilibria are locally unique. ■

#### 3.A.3.3 Proof of Proposition 3.2

The proof follows the same logic as the proof of Proposition 3.1. Consider a given $(\hat{r}^1, \hat{r}^2) \in \mathcal{E}$ in the laissez-faire ($g = 0$) and the corresponding consumption allocation $C_{LF} = (c^1_{LF}, x^1_{LF}, c^2_{LF}, x^2_{LF})$, where $c^i_{LF} := c^i_d(y^i, \hat{r}^i)$ and $x^i_{LF} := x^i_d(y^i, \hat{r}^i)$.

Set the public provision level to $g_r = x^1_{LF}$. By the assumption that reference levels do not jump, policy $g_r$ leads to the same consumption allocation $C_{LF}$ as in the laissez-faire; as $g_r = x^1_{LF}$, by demand functions (3.18) and (3.19), individuals of type 1 choose $c^1(g_r, \hat{r}^1) = y^1 - g_r = c^1_{LF}$ and $x^1(g_r, \hat{r}^1) = g_r = x^1_{LF}$. Since $x^1_d(y^1, r^1) < x^2_d(y^2, r^2)$ by assumption, we have $g_r < x^2_{LF}$. Hence, individuals of type 2 choose $c^2(g_r, \hat{r}^2) = c^2_d(y^2, \hat{r}^2) = c^2_{LF}$ and $x^2(g_r, \hat{r}^2) = c^2_d(y^2, \hat{r}^2) = x^2_{LF}$. Therefore, at $g_r$, individuals of type 1 [type 2] are [not] crowded out by public provision, and system (3.20) reads

$$\begin{aligned} &r^1 - h^1(y^1 - g, c^2_{LF}) = 0 \\ &r^2 - h^2(c^2_{LF}, y^1 - g) = 0. \end{aligned} \tag{3.44}$$

Given $(\hat{r}^1, \hat{r}^2)$, indirect utilities can be expressed as

$$V^1(g) = U(y^1 - g, g, \Delta^1(y^1 - g, \hat{r}^1(g))), \tag{3.45}$$

$$V^2(g) = U(c^2_d(y^2, \hat{r}(g)), x^2_d(y^2, \hat{r}(g)), \Delta^2(c^2_d(y^2, \hat{r}(g)), \hat{r}^2(g))). \tag{3.46}$$

Consider a change in the public provision level from $g_r$ to $g_p = g_r + dg$, where $dg > 0$ and $dg \to 0$. Using (3.44), the effect of this policy change on both types' reference levels is

$$\frac{d\hat{r}^1}{dg} = -\frac{\partial h^1}{\partial c^1} - \frac{1}{D_1}\frac{\partial h^2}{\partial c^1}\frac{\partial h^1}{\partial c^2}\frac{\partial c^2_d}{\partial r^2} \tag{3.47}$$

$$\frac{d\hat{r}^2}{dg} = -\frac{1}{D_1}\frac{\partial h^2}{\partial c^1}, \tag{3.48}$$

where

$$D_1 = 1 - \frac{\partial h^2}{\partial c^2}\frac{\partial c^2_d}{\partial r^2} > 0$$

by Assumption 3.1. Using (3.47), (3.48) and the first-order conditions of the individual maximization problems (3.40), the change in indirect utilities induced by the change from $g_r$ to $g_p$ can be calculated to

$$\frac{dV^1}{dg} = -U^1_\Delta \frac{\partial \Delta^1}{\partial r^1}\left[\frac{\partial h^1}{\partial c^1} + \frac{1}{D_1}\frac{\partial h^2}{\partial c^1}\frac{\partial h^1}{\partial c^2}\frac{\partial c^2_d}{\partial r^2}\right], \tag{3.49}$$

$$\frac{dV^2}{dg} = -U^2_\Delta \frac{1}{D_2}\frac{\partial h^2}{\partial c^1}, \tag{3.50}$$

where $U^i_\Delta := \partial U(c^i, x^i, \Delta^i)/\partial \Delta$ and (3.49) and (3.50) are evaluated at $g_r$. Under KUJ,

$\partial c_d^2/\partial r^2 > 0$. Hence, $dV^1/dg$ and $dV^2/dg$ are non-negative at $g_r$. If $\partial h^1/\partial c^1 > 0$ or $\partial h^2/\partial c^1 > 0$, we have $dV^1 > 0$ or $dV^2 > 0$.

To complete the proof, we have to show that individuals of type 1 [type 2] are still [not] crowded out by public provision after the policy change from $g_r$ to $g_p$. As demand functions are continuous and we require $x_{LF}^2 > x_{LF}^1$, individuals of type 2 are not crowded out by public provision at $g_p$. Type 1 individuals remain crowded out if the difference $\Omega^1(g, r^1) := g - x_d^1(y^1, r^1)$ is greater or equal to zero at $g_p$ (see demand functions (3.18) and (3.19)). This holds if

$$d\Omega^1(g, r^1) = dg - \frac{\partial x_d^1}{\partial r^1} dr^1 \geq 0, \tag{3.51}$$

at $g_r$. Dividing by $dg$ and inserting (3.47), this is equivalent to

$$-\frac{\partial x_d^1}{\partial r^1}\frac{\partial h^1}{\partial c^1} - \frac{1}{D_1}\frac{\partial h^2}{\partial c^1}\frac{\partial h^1}{\partial c^2}\frac{\partial c^2}{\partial r^2}\frac{\partial x_d^1}{\partial r^1} \leq 1. \tag{3.52}$$

From individuals' budget constraints, KUJ implies $\partial x^i/\partial r^i = -\partial c^i/\partial r^i < 0$. Hence, (3.52) can be rewritten

$$\frac{\partial h^1}{\partial c^1}\frac{\partial c_d^1}{\partial r^1} + \frac{1}{D_1}\frac{\partial h^2}{\partial c^1}\frac{\partial h^1}{\partial c^2}\frac{\partial c_d^2}{\partial r^2}\frac{\partial c_d^1}{\partial r^1} \leq 1. \tag{3.53}$$

By item (i) of Assumption 3.1,

$$\left(1 - \frac{\partial h^1}{\partial c^1}\frac{\partial c_d^1}{\partial r^1}\right)\left(1 - \frac{\partial h^2}{\partial c^2}\frac{\partial c_d^2}{\partial r^2}\right) - \frac{\partial h^1}{\partial c^2}\frac{\partial c_d^2}{\partial r^2}\frac{\partial h^2}{\partial c^1}\frac{\partial c_d^1}{\partial r^1} > 0. \tag{3.54}$$

Dividing by $D_1$ and rearranging leads to (3.53). Hence, individuals of type 1 stay crowded out after the change to policy $P_p$. This proves Proposition 3.2. ■

### 3.A.4 Derivation of optimal consumption taxes in the absence of public provision

For a given policy $P = (b^1, b^2, g, t)$, each individual's budget set is given by $B_t^i = \{(c^i, x^i) : (1+t)\, c^i + x^i - g \leq b^i, x^i \geq g\}$. The unique solution to the individual maximization problem (3.6) must satisfy the first-order conditions

$$MRS(c^i, x^i) - p \geq 0 \quad \text{and} \quad \left(x^i - g\right)\left[MRS(c^i, x^i) - p\right] = 0. \tag{3.55}$$

Demand functions for goods $c$ and $x$ can be expressed as follows:

$$c^i(b^i, g, p) = \begin{cases} c_d(b^i + g, p) & \text{if } g < x_d(b^i + g, p), \\ b^i/p & \text{if } g \geq x_d(b^i + g, p), \end{cases} \tag{3.56}$$

$$x^i(b^i, g, p) = \begin{cases} x_d(b^i + g, p) & \text{if } g < x_d(b^i + g, p), \\ g & \text{if } g \geq x_d(b^i + g, p). \end{cases} \tag{3.57}$$

The functions $c_d(I, p)$ and $x_d(I, p)$ again give individuals' ordinary or unconstrained demands, and solve problem (3.6) for $B_u := \{(c, x) : pc + x \leq I\}$. Using that $p = p(t)$, we define indirect utility of type $i$ as

$$\begin{aligned} V^i(b^i, b^j, g, p(t)) := {} & u^i(c^i(b^i, g, p(t)), x^i(b^i, g, p(t))) + c^i(b^i, g, p(t)) \\ & - h^i(c^i(b^i, g, p(t)), c^j(b^j, g, p(t))). \end{aligned} \tag{3.58}$$

Optimal policies in the absence of public provision solve

$$\max_{b^1, b^2, t} \quad V^1(b^1, b^2, 0, p(t)) \quad \text{s.t.} \tag{3.59}$$

$$\begin{aligned} \text{(i)}: & \ V^2(b^1, b^2, 0, p(t)) \geq \bar{U}^2, \\ \text{(ii)}: & \ y^1 - b^1 + tc^1 + y^2 - b^2 + tc^2 \geq 0. \end{aligned}$$

We define $\mathcal{S}$ as the set of policies $P_0 = (b_0^1, b_0^2, 0, t_0)$ such that there exists a level $\bar{U}^2$ and the policy solves (3.59).

Denote the Lagrangian to problem (3.59) by

$$\begin{aligned} L = {} & V^1(b^1, b^2, 0, p(t)) + \mu \left[ V^2(b^1, b^2, 0, p(t)) - \bar{U}^2 \right] \\ & + \lambda \left[ y^1 - b^1 + tc^1 + y^2 - b^2 + tc^2 \right], \end{aligned} \tag{3.60}$$

where $\mu$ and $\lambda$ are the Lagrange multipliers associated with constraints (i) and (ii) of problem (3.59). Any interior solution $P_0 = (b_0^1, b_0^2, 0, t_0)$ in $\mathcal{S}$ must satisfy the first-order conditions:

$$\frac{\partial L}{\partial b^1} = \frac{1}{p_0}(u_c^1 + 1) - \frac{\partial h^1}{\partial c^1}\frac{\partial c^1}{\partial b^1} - \mu^0 \frac{\partial h^2}{\partial c^1}\frac{\partial c^1}{\partial b^1} + \lambda^0 \left(-1 + t_0 \frac{\partial c^1}{\partial b^1}\right) = 0 \tag{3.61}$$

$$\frac{\partial L}{\partial b^2} = -\frac{\partial h^1}{\partial c^2}\frac{\partial c^2}{\partial b^2} + \mu^0 \left[\frac{1}{p_0}(u_c^2 + 1) - \frac{\partial h^2}{\partial c^2}\frac{\partial c^2}{\partial b^2}\right] + \lambda^0 \left(-1 + t_0 \frac{\partial c^2}{\partial b^2}\right) = 0 \tag{3.62}$$

$$\begin{aligned}\frac{\partial L}{\partial t} &= -\frac{1}{p_0}(u_c^1 + 1)\, c^1 - \frac{\partial h^1}{\partial c^1}\frac{\partial c^1}{\partial p} - \frac{\partial h^1}{\partial c^2}\frac{\partial c^2}{\partial p} \\ &+ \mu^0 \left[-\frac{1}{p_0}(u_c^2 + 1)\, c^2 - \frac{\partial h^2}{\partial c^1}\frac{\partial c^1}{\partial p} - \frac{\partial h^2}{\partial c^2}\frac{\partial c^2}{\partial p}\right] \\ &+ \lambda^0 \left[c^1 + t_0 \frac{\partial c^1}{\partial p} + c^2 + t_0 \frac{\partial c^2}{\partial p}\right] = 0,\end{aligned} \tag{3.63}$$

where $p_0 = 1 + t_0$. Solving (3.61) and (3.62) for $\mu^0$ and $\lambda^0$ yields:

$$\mu^0 = \frac{1}{D_2}\left[\frac{\left(-1 + t_0 \partial c^2/\partial b^2\right)}{-1 + t_0 \partial c^1/\partial b^1}\left[\frac{1}{p_0}(u_c^1 + 1) - \frac{\partial h^1}{\partial c^1}\frac{\partial c^1}{\partial b^1}\right] + \frac{\partial h^1}{\partial c^2}\frac{\partial c^2}{\partial b^2}\right] \tag{3.64}$$

$$\begin{aligned}\lambda^0 = &-\frac{1}{\left(-1 + t_0 \partial c^1/\partial b^1\right)}\frac{1}{D_2}\left[\frac{1}{p_0}(u_c^2 + 1)\left[\frac{1}{p_0}(u_c^1 + 1) - \frac{\partial h^1}{\partial c^1}\frac{\partial c^1}{\partial b^1}\right]\right. \\ &\left.-\frac{1}{p_0}(u_c^1 + 1)\frac{\partial h^2}{\partial c^2}\frac{\partial c^2}{\partial b^2} + \frac{\partial c^1}{\partial b^1}\frac{\partial c^2}{\partial b^2}\left(\frac{\partial h^1}{\partial c^1}\frac{\partial h^2}{\partial c^2} - \frac{\partial h^2}{\partial c^1}\frac{\partial h^1}{\partial c^2}\right)\right],\end{aligned} \tag{3.65}$$

where

$$D_2 := \left[\frac{1}{p_0}(u_c^2 + 1) - \frac{\partial h^2}{\partial c^2}\frac{\partial c^2}{\partial b^2}\right] + \frac{\left(-1 + t_0 \partial c^2/\partial b^2\right)}{-1 + t_0 \partial c^1/\partial b^1}\frac{\partial h^2}{\partial c^1}\frac{\partial c^1}{\partial b^1} \tag{3.66}$$

Combining (3.61), (3.62) with (3.63), substituting for $\lambda^0$ and $\mu^0$, and rearranging gives:

$$t_0 = \frac{\Gamma^1 \beta + \Gamma^2 \gamma}{\beta + \gamma}, \tag{3.67}$$

where

$$\beta := \left[\frac{\partial \tilde{c}^1}{\partial p}\left(1 - \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^2}\frac{\partial c^2}{\partial b^2}\right) + \frac{\partial \tilde{c}^2}{\partial p}\frac{1}{u_x^1}\frac{\partial h^1}{\partial c^2}\frac{\partial c^1}{\partial b^1}\right] < 0, \tag{3.68}$$

$$\gamma := \left[\frac{\partial \tilde{c}^{\,2}}{\partial p}\left(1 - \frac{1}{u_x^1}\frac{\partial h^1}{\partial c^1}\frac{\partial c^1}{\partial b^1}\right) + \frac{\partial \tilde{c}^{\,1}}{\partial p}\frac{1}{u_x^2}\frac{\partial h^2}{\partial c^1}\frac{\partial c^2}{\partial b^2}\right] < 0. \tag{3.69}$$

In (3.68) and (3.69), $\tilde{c}^{\,i}\left(p, g, \bar{v}^i\right)$ denotes individual $i$'s Hicksian or compensated demand function for good $c$ for a given utility level $\bar{v}^i$.[37] The assumption that $U_*^i$ increases in $c^i$ implies that the terms in parentheses in (3.68) and (3.69) are, respectively, positive. Since $\partial \tilde{c}^{\,i}/\partial p < 0$, we have $\beta < 0$ and $\gamma < 0$. As $\Gamma^i < 0$ for at least one income type, $t_0 > 0$. Defining

$$\alpha := \frac{\beta}{\beta + \gamma} > 0$$

(3.67) can be written as

$$t_0 = \left[\alpha \Gamma^1 + (1-\alpha)\,\Gamma^2\right] > 0,$$

which coincides with (3.23).

### 3.A.5 Proof of Proposition 3.3

Consider an optimal policy $P_0 \in \mathcal{S}$. If $\Gamma^1|_{C=C_0} = \Gamma^2|_{C=C_0}$ at $P_0$, (3.23) implies $t_0 = \Gamma^1|_{C=C_0} = \Gamma^2|_{C=C_0}$. By (3.55), individuals choose $c$ and $x$ such that $MRS(c_0^i, x_0^i) = 1 + t_0$. Substituting for $t_0$ gives $MRS(c_0^i, x_0^i) - \Gamma^i|_{C=C_0} = 1$ for $i = 1, 2$, which coincides with (3.7). As the utility function $U_*^i$ is strongly quasi-concave, any allocation that satisfies (3.7) is Pareto efficient. Conversely, if $\Gamma^1|_{C=C_0} \neq \Gamma^2|_{C=C_0}$ at $P_0$, we have $MRS(c^i, x^i) - \Gamma^i \neq 1$ for at least one type, and condition (3.7) does not hold. Hence, policy $P_0$ implements an efficient allocation if and only if $\Gamma^1|_{C=C_0} = \Gamma^2|_{C=C_0}$. ■

### 3.A.6 Proof of Proposition 3.4

The proof follows the same logic as the proof of Proposition 3.1. Consider a given policy $P_0 \in \mathcal{S}$. We will show that if $\Gamma^1 > \Gamma^2$ at $P_0$, there exists a feasible policy that achieves

[37] Formally, $\tilde{c}^{\,i}\left(p, g, \bar{v}\right)$ are obtained from the expenditure minimization problem

$$\min_{c^i, x^i} \quad p \cdot c^i + x^i - g \quad \text{s.t.} \quad u^i(c^i, x^i) + c^i - r^i \geq \bar{v}^i, \qquad x^i \geq g, \quad c^i \geq 0. \tag{3.70}$$

Given our assumptions on preferences, there exists a unique $\tilde{c}^{\,i}(p, g, \bar{v}^i)$ for every $(p, g, \bar{v}^i)$.

a Pareto improvement over $P_0$.

We first prove that there exists a feasible policy with $g > 0$ which leads to the same consumption allocation $C_0 = (c_0^1, x_0^1, c_0^2, x_0^2)$ as $P_0 = (b_0^1, b_0^2, 0, t_0)$. Specifically, consider the policy $P_r$ with

$$b_r^1 = b_0^1 - g_r, \quad b_r^2 = b_0^2 - g_r, \quad g_r = x_0^1, \quad t_r = t_0. \tag{3.71}$$

At $P_r$, we have $x_d(b_r^1 + g_r, p_r) = x_d(b_0^1, p_0) = x_0^1$ and $x_d(b_r^2 + g_r, p_r) = x_d(b_0^2, p_0) = x_0^2$. Since $g_r = x_0^1 < x_0^2$, by (3.56) and (3.57), it follows that types 1 and 2 respectively choose $c^1(b_r^1, g_r, p_r) = b_r/p_r = c_0^1$, $x^1(b_r^1, g_r, p_r) = g_r = x_0^1$, $c^2(b_r^2, g_r, p_r) = c_d(b_0^2, p_0) = c_0^2$ and $x^2(b_r^2, g_r, p_r) = x_d(b_0^2, p_0) = x_0^2$. Thus, consumption allocations and individuals' indirect utilities at $P_0$ and $P_r$ coincide.

Substitution of policy $P_r$ into the government budget constraint (3.22) gives

$$y^1 - b_0^1 + t_0 c_0^1 + y^2 - b_0^2 + t_0 c_0^2 = 0. \tag{3.72}$$

As (3.72) must hold at $P_0$, policy $P_r$ is feasible. Since individuals of type 1 [type 2] are [not] constrained by public provision at $P_r$, we can express indirect utilities and the government budget as

$$\begin{aligned}
V^1(b^1, b^2, g, p) =& u(b^1/p, g) + b^1/p - h^1(b^1/p, c_d(b^2 + g, p)), && (3.73)\\
V^2(b^2, b^1, g, p) =& u(c_d(b^2 + g, p), x_d(b^2 + g, p)) && (3.74)\\
&+ c_d(b^2 + g, p) - h^2(c_d(b^2 + g, p), b^1/p), \\
G =& y^1 - b^1 + tb^1/p + y^2 - b^2 + tc_d(b^2 + g, p) - 2g. && (3.75)
\end{aligned}$$

Now, consider a change from $P_r$ to policy $P_p = (b_r^1 + db^1, b_r^2 + db^2, g_r + dg, p_0)$, where $dg > 0$ and $dg \to 0$. Net incomes are adjusted such that (i) the government budget $G$ remains balanced and (ii) the indirect utility of type 2 does not change. The consumption tax is held constant at $t = t_0$. Using (3.74) and (3.75), requirements (i) and (ii) can be represented by

$$dV^2 = -\frac{\partial h^2}{\partial c^1}\frac{1}{p}db^1 + \left(u_x^2 - \frac{\partial h^2}{\partial c^2}\frac{\partial c_d^2}{\partial I}\right)db^2 + \left(u_x^2 - \frac{\partial h^2}{\partial c^2}\frac{\partial c_d^2}{\partial I}\right)dg = 0, \tag{3.76}$$

$$dG = -\frac{1}{p}db^1 + \left(-1 + t\frac{\partial c_d^2}{\partial I}\right)db^2 + \left[-1 + \left(-1 + t\frac{\partial c_d^2}{\partial I}\right)\right]dg = 0, \tag{3.77}$$

where we abbreviated $\partial c_d^i/\partial I := \partial c_d(b^i + g, p)/\partial I$ and used that $(u_c^2 + 1)/u_x^2 = p$ at policy $P_r$. Solving for $db^1$ and $db^2$ gives

$$db^1 = -\left[1 - \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^2}\frac{\partial c_d^2}{\partial I}\right](1+t)\,\frac{1}{D_3}dg \tag{3.78}$$

$$db^2 = -\left[1 - \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^2}\frac{\partial c_d^2}{\partial I} + \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^1} - \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^1}\left(-1 + t\frac{\partial c_d^2}{\partial I}\right)\right]\frac{1}{D_3}dg, \tag{3.79}$$

with

$$D_3 := \left[\underbrace{1 - \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^2}\frac{\partial c_d^2}{\partial I}}_{>0}\underbrace{-\frac{1}{u_x^2}\frac{\partial h^2}{\partial c^1}\left(-1 + t\frac{\partial c_d^2}{\partial I}\right)}_{>0}\right] > 0. \tag{3.80}$$

The first term in (3.80) is positive as $U_*^i$ increases in $c^i$. The same holds for the second term since $(-1 + t\partial c_d^2/\partial I) < 0$.[38]
The policy change from $P_r$ to $P_p$ achieves a Pareto improvement if the indirect utility of type 1 increases. Using (3.73), we can represent the change in $V^1$ by

$$dV^1 = u_x^1\left[\left(1 - \frac{1}{u_x^1}\frac{\partial h^1}{\partial c^1}\frac{1}{p}\right)db^1 - \frac{1}{u_x^1}\frac{\partial h^1}{\partial c^2}\frac{\partial c_d^2}{\partial I}db^2 + \left(1 - \frac{1}{u_x^1}\frac{\partial h^1}{\partial c^2}\frac{\partial c_d^2}{\partial I}\right)dg\right], \tag{3.81}$$

where we used that $(u_c^1 + 1)/u_x^1 = p$ and factored out $u_x^1$. Making use of (3.78), (3.79) and the definitions of $\Gamma^1$ and $\Gamma^2$, (3.81) can be rewritten as

$$dV^1 = u_x^1\left[-\left(-\Gamma^1 + t\right)\left[1 - \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^2}\frac{\partial c_d^2}{\partial I}\right] - \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^1}\frac{\partial c_d^2}{\partial I}\left(-\Gamma^2 + t\right)\right]\frac{1}{D_3}dg. \tag{3.82}$$

Substitution of the expression for the optimal tax in the absence of public provision (3.23) in (3.82), and taking into account that consumption allocations at $P_0$ and $P_r$ coincide, yields

$$dV^1 = -u_x^1\left[\underbrace{\frac{\partial \tilde{c}^2}{\partial p}\frac{1}{\beta+\gamma}}_{>0}\left[\Gamma^1 - \Gamma^2\right]D_4\right]\frac{1}{D_3}dg, \tag{3.83}$$

[38] From individuals' budget constraint, $c_d(I^2, p) = 1/p(I^2 - x_d(I^2, p))$. Differentiating with respect to $I$ an rearranging gives

$$-1 + t\frac{\partial c_d^2}{\partial I} + \frac{\partial c_d^2}{\partial I} + \frac{\partial x_d^2}{\partial I} = 0.$$

As both goods are normal, we must have $(-1 + t\partial c_d^2/\partial I) < 0$.

where

$$D_4 := \left[ -1 + \frac{1}{u_x^1} \frac{\partial h^1}{\partial c^1} \frac{\partial c_d^1}{\partial I} + \frac{1}{u_x^2} \frac{\partial h^2}{\partial c^2} \frac{\partial c_d^2}{\partial I} \left( 1 - \frac{1}{u_x^1} \frac{\partial h^1}{\partial c^1} \frac{\partial c_d^1}{\partial I} \right) \right.$$
$$\left. + \frac{1}{u_x^1} \frac{\partial h^1}{\partial c^2} \frac{\partial c_d^1}{\partial I} \frac{1}{u_x^2} \frac{\partial h^2}{\partial c^1} \frac{\partial c_d^2}{\partial I} \right] < 0$$

The sign of $D_4$ follows since, at the optimal policy $P_0$, the Lagrange multiplier associated with the government budget constraint, $\lambda^0$, must be larger than zero. Thus, the proposed policy achieves a Pareto improvement, i.e., $dV^1 > 0$, if $\Gamma^1 > \Gamma^2$ at $P_0$.
To complete the proof, we must show that individuals of type 1 [type 2] indeed remain [not] crowded out by public provision if we change the policy from $P_r$ to $P_p$. By the continuity of demand functions, this is satisfied for type 2. Individuals of type 1 are crowded out at $P_p$ if the difference $\Omega^1(b^1, g, p) := g - x_d^1(b^1 + g, p)$ is greater or equal to zero after the change to policy $P_p$. Formally, at $P_r$, this requires

$$\frac{\partial \Omega^1(b^1, g, p)}{\partial b^1} db^1 + \frac{\partial \Omega^1(b^1, g, p)}{\partial g} dg = \underbrace{\left[ 1 - \frac{\partial x_d^1}{\partial I} \right] dg}_{>0} \underbrace{- \frac{\partial x_d^1}{\partial I} db^1}_{>0} > 0. \tag{3.84}$$

The first term of the right hand side of (3.84) is positive as both goods are normal. The sign of the second term follows since $db^1 < 0$ by (3.78). Hence, type 1 individuals are crowded out at $P_p$. As similar arguments apply for all $P_0 \in \mathcal{S}$, this finishes the proof of Proposition 3.4. ■

### 3.A.7 Example 2

Assume that the sub-utility function $u$ and reference functions $h^i$ are respectively given by

$$\tilde{U}^i(c, x, r^i) = \frac{1}{1-\sigma} \left( c^{1-\sigma} + \delta x^{1-\sigma} \right) - r^i, \quad h^1(c^1, c^2) = c^2, \quad h^2(c^2, c^1) = 0.$$

Hence, $\Gamma^2 > \Gamma^1 = 0$. We choose parameters $y^1 = 10$, $y^2 = 15$, $\sigma = 0.8$ and $\delta = 0.4$. Setting $\bar{U}^2 = 10.7189$ to laissez-faire level enjoyed by the rich, the optimal tax in the absence of public provision is $t_0 = 1.04$, and the poor obtain indirect utility $V^1 = -0.35$. Now, consider a switch to a policy $P' = (b^1, b^2, g, 0)$, where $b^1 = y^1 - 1.1g$, $b^2 = y^2 - 0.9g$ and $g = 6.05$. At this policy, both types are constrained by public provision and obtain utilities $V^1 = -0.32$ and $V^2 = 10.7193$. Hence, under policy $P'$, the poor and the rich are better off compared to $P_0$.

### 3.A.8 Proof of Proposition 3.5

Consider a given Pareto efficient allocation $C_* \in \mathcal{P}$.

**Item (i):** To prove the "if"-part, consider the policy $P_* = (b^1_*, b^2_*, g_*)$, with

$$b^1_* = c^1_*, \quad b^2_* = c^2_* + x^2_* - g_*, \quad g_* = x^1_*. \tag{3.85}$$

At policy $P_*$, individuals of type 1 choose the bundle $(c^1_*, x^1_*)$ if they are (just) constrained by public provision and do not buy additional units of $x$ on the market. This happens if and only if

$$MRS(c^1_*, x^1_*) \geq 1. \tag{3.86}$$

From condition (3.7) we know that $MRS(c^1_*, x^1_*) = 1 + \Gamma^1$. Hence, as $\Gamma^1 > 0$, condition (3.86) holds. Individuals of type 2 can afford their intended bundle: by (3.85), their disposable income is effectively $I^2_* = b^2_* + g_*$ which is sufficient to buy $(c^2_*, x^2_*)$. Moreover, since we consider only $C_* \in \mathcal{P}$ where $x^1_* < x^2_*$, by demand functions (3.12) and (3.13), we have $g < x_d(I^2_*) = x^2_*$ such that individuals of type 2 are not constrained at $P_*$. Hence, $(c^2_*, x^2_*)$ is the optimal choice: utility-maximization requires $MRS(c^2, x^2) = 1$. By (3.7) and strict convexity of preferences, this condition can only hold at $(c^2_*, x^2_*)$.
Substituting $P_*$ into the government budget constraint (3.10) gives

$$c^1_* + x^1_* + c^2_* + x^2_* - y^1 - y^2 = 0, \tag{3.87}$$

which must hold for any $C_*$ and therefore proves feasibility.
To prove the "only if" part, note that if $\Gamma^2 > 0$ policy $P_*$ can never implement $C_*$: as individuals of type 2 choose goods $c$ and $x$ according to $MRS(c^2, x^2) = 1$ and $MRS(c^2_*, x^2_*) = 1 + \Gamma^2$ by (3.7), they would always choose a consumption bundle different from $(c^2_*, x^2_*)$ when income is $b^2_*$.
Further, there exist no other policies that might lead to $C_*$. Implementation with policies where $g > x^1_*$ is impossible since, by monotonicity, individuals never forego the publicly provided amount $g$. Hence, at least one income type would not consume the efficient level of good $x$. We can also rule out policies where $g < x^1_*$: if one or both types

are constraint, Pareto efficiency cannot be attained when $x_*^1 < x_*^2$; if both individuals are not crowded out, they would top-up to a bundle that is not Pareto efficient, since both choose goods $c$ and $x$ such that $MRS\left(c^i, x^i\right) = 1$.

**Item (ii):** Consider the policy $P_* = (b_*^1, b_*^2, g_*, t_*)$ with

$$b_*^1 = (1+t_*)c_*^1, \quad b_*^2 = (1+t_*)\,c_*^2 + x_*^2 - g_*, \quad g_* = x_*^1, \quad t_* = \Gamma^2\big|_{C=C_*}. \tag{3.88}$$

We first show that individuals choose $C_*$ under $P_*$ if and only if $\Gamma^1 > \Gamma^2$. We then verify that $P_*$ is feasible if individuals were to choose $C_*$ at this policy. In a last step, we proof that $P_*$ is the only policy that can achieve $C_*$.
To see that $C_*$ is consistent with utility maximization, consider first the decision problem of type 1 individuals. The bundle $(c_*^1, x_*^1)$ is affordable since $g = x_*^1$ and the net income $b_*^1$ is designed such that $c_*^1$ can be just attained if $x^1 = g$. It is optimal for type 1 to choose $(c_*^1, x_*^1)$ if

$$MRS(c_*^1, x_*^1) \geq p = 1 + t_*. \tag{3.89}$$

Substituting for $t_*$ in (3.89) and rearranging gives $MRS(c_*^1, x_*^1) - \Gamma^2 \geq 1$. From condition (3.7) we know that $MRS(c_*^1, x_*^1) = 1 + \Gamma^1$. Thus, $(c_*^1, x_*^1)$ is optimal for individuals of type 1 if and only if $\Gamma^1 \geq \Gamma^2$.
The rich can also afford their intended bundle: by (3.88), their disposable income is effectively $I_*^2 = b_*^2 + g_*$ which is sufficient to buy $(c_*^2, x_*^2)$. Moreover, since we consider only $C_* \in \mathcal{P}$ where $x_*^1 < x_*^2$, we have $g < x_*^2$, and the rich are not constrained at $P_*$. Hence, $(c_*^2, x_*^2)$ is the optimal choice: utility-maximization requires $MRS(c^2, x^2) = p = 1 + t_*$. As $t_* = \Gamma^2$, by (3.7) this condition can only hold at $(c_*^2, x_*^2)$.
Inserting $P_*$ into the government budget constraint (3.22) gives

$$c_*^1 + x_*^1 + c_*^2 + x_*^2 - y^1 - y^2 = 0, \tag{3.90}$$

which must hold for any $C_*$ and therefore proves feasibility.
$P^*$ is the only policy that may support $C_*$. Implementation with policies where $g > x_*^1$ is impossible since, by monotonicity, individuals never forego the publicly provided amount $g$. Hence, at least one income type would not consume the efficient level of good $x$. We can also rule out policies where $g < x_*^1$: if one or both types are constraint, Pareto efficiency cannot be attained when $x_*^1 < x_*^2$; if both individuals are not crowded

out, they would top-up to a bundle that is not Pareto efficient, since in the case where $\Gamma^1 \neq \Gamma^2$, the uniform tax rate can never be set such that private optimization yield $MRS + \Gamma^i = 1$ for $i = 1, 2$ for any given net incomes. ■

### 3.A.9 Proof of Proposition 3.6

**Item (i):** If consumption taxes are not feasible, $t = 0$ and $p = 1$. Consider a switch from the laissez-faire to the following policy $P_r$:

$$e_r = c^1_{LF}; \quad b^1_r = y^1 - e_r; \quad b^2_r = y^2. \tag{3.91}$$

This policy induces individuals to choose the same consumption bundles as in the laissez-faire. Since $\hat{e}(b^1_r, p) < c_d(b^1_r, p) < c_d(y^1, p) = c^1_{LF}$, type 1 individuals choose public provision at $P_r$. Hence, by (3.27) and (3.28), $c^1(b^1_r, p, e_r) = e_r = c^1_{LF}$ and $x^1(b^1_r, p, e_r) = b^1_r = x^1_{LF}$. By the assumption $c^1_{LF} < \hat{e}^2(y^2, p)$, individuals of type 2 opt out and, as $b^2_r = y^2$, select $c^2(b^2_r, p, e_r) = c^2_{LF}$ and $x^2(b^2_r, p, e_r) = x^2_{LF}$, respectively. Policy $P_r$ is also feasible: substituting it into the government budget constraint (see Definition 3.2) gives

$$y^1 - c^1_{LF} - x^1_{LF} + y^2 - c^2_{LF} - x^2_{LF} = 0,$$

which holds since $C_{LF}$ satisfies the economy's resource constraint.
Now, marginally decrease $e$ at $P_r$ and finance it by $de = -db^1$. By the continuity of utility functions, type 2 stays out of the public system after this policy change. The effect on both types' indirect utilities is

$$d\tilde{V}^1_{in} = -\frac{\partial h^1}{\partial c^1} de, \tag{3.92}$$

$$d\tilde{V}^2_{out} = -\frac{\partial h^2}{\partial c^1} de. \tag{3.93}$$

Hence, decreasing $e$ benefits at least one type if $\partial h^1 / \partial c^1 > 0$ or $\partial h^2 / \partial c^1 > 0$.

**Item (ii):** If a tax on the positional good is available, the benchmark is policy $P_0 = (b^1_0, b^2_0, 0, t_0)$. Consider a change from $P_0$ to the alternative policy $P_r$ with

$$e_r = c^1_0, \quad b^1_r = b^1_0 - p_0 e_r, \quad b^2_r = b^2_0, \quad t_r = t_0. \tag{3.94}$$

At $P_r$, type 1 chooses the publicly provided amount $e$: as $b_0^1 > b_r^1$, we have $\hat{e}(b_r^1, p_0) < c_d(b_r^1, p_0) < c_d(b_0^1, p_0) = c_0^1$. Thus, type 1 opts in when $e = c_0^1$. By assumption ($e = c_0^1 < \hat{e}(b_0^2, p_0)$), individuals of type 2 opt out. Consequently, both types choose the same consumption bundle as in the situation without public provision and are thus equally well-off. Further, we have $N = 1$, and the proposed policy satisfies the government budget constraint.

We now show that if $\Gamma^1 \neq \Gamma^2$ at $P_0$, a Pareto-improving policy always exists. To see this, marginally change $e$ by $de$ and adjust net incomes $b^i$ such that (i) the government budget remains balanced and (ii) utility type 2 individuals $\tilde{V}_{out}^2$ does not change. Since $de \to 0$, the continuity of utility functions ensures that type 1 (type 2) individuals are not induced to opt out (in) after the policy change. Formally, the two conditions (i) and (ii) require

$$- db^1 + \left(-1 + t\frac{\partial c^2}{\partial b^2}\right) db^2 - de = 0 \tag{3.95}$$

$$\left(u_x^2 - \frac{\partial h^2}{\partial c^2}\frac{\partial c^2}{\partial b^2}\right) db^2 - \frac{\partial h^2}{\partial c^1} de = 0, \tag{3.96}$$

where we used that $\partial c^1/\partial b^1 = 0$, $\partial c^1/\partial e = 1$, $\partial \tilde{V}_{out}^2/\partial e = 0$ at $P_r$. Solving for $db^1$ and $db^2$ yields

$$db^1 = \frac{\left(-1 + t\frac{\partial c^2}{\partial b^2}\right)\frac{1}{u_x^2}\frac{\partial h^2}{\partial c^1} - \left[1 - \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^2}\frac{\partial c^2}{\partial b^2}\right]}{1 - \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^2}\frac{\partial c^2}{\partial b^2}} de \tag{3.97}$$

$$db^2 = \frac{\frac{1}{u_x^2}\frac{\partial h^2}{\partial c^1}}{1 - \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^2}\frac{\partial c^2}{\partial b^2}} de \tag{3.98}$$

The above policy change achieves a Pareto improvement if the utility of type 1 increases. Total differentiation of $\tilde{V}_{in}^1$, combined with $\partial c^1/\partial b^1 = 0$, $\partial c^1/\partial e = 1$, $\partial \tilde{V}_{out}^2/\partial e = 0$ at $P_r$, (3.97) and (3.98) gives

$$d\tilde{V}_{in}^1 = u_x^1 \left[-\Gamma^1 + t\right] de + \frac{u_x^1}{1 - \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^2}\frac{\partial c^2}{\partial b^2}} \left[-\Gamma^2 + t\right] \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^1}\frac{\partial c^2}{\partial b^2} de. \tag{3.99}$$

Combining (3.99) with the optimal commodity tax $t_0$ yields

$$d\tilde{V}_{in}^{1} = \frac{u_x^1}{\left[1 - \frac{1}{u_x^2}\frac{\partial h^2}{\partial c^2}\frac{\partial c^2}{\partial b^2}\right]} \underbrace{\frac{\partial \tilde{c}^{\,2}}{\partial p}\frac{1}{\beta + \gamma}}_{>0} \left[\Gamma^1 - \Gamma^2\right] D_4\, de, \tag{3.100}$$

where $D_4 < 0$ is defined as in the proof of Proposition 3.4. Thus, if $\Gamma^1 < (>)\, \Gamma^2$, a marginal increase (decrease) in $e$ from $P_r$ yields a Pareto improvement over policy $P_0$. This proves item (ii) of the proposition. ■

# Chapter 4

## Present-Bias, Price Subsidies, and Public Provision

## 4.1 Introduction

When making intertemporal decisions, many people are biased towards the present: they assign higher relative weight to well-being in the current period than they did if asked in any period before (O'Donoghue and Rabin, 2001).[1] Such present-biased preferences are time-inconsistent and capture the idea that people may have self-control problems: due to a tendency to take immediate rewards and to avoid immediate costs, people's actions may deviate from what they had originally planned.

Self-control problems have important implications for the consumption of essential goods like education, health care, or old-age consumption. A common feature of these goods is that they involve up front costs but sizeable parts of their benefits arise in the future. For example, investments in education require considerable current expenses in exchange for an increased prospective earnings potential. Likewise, taking preventive medication, routine check-ups, medical advice, or a proper diet – although offering some immediate benefits – mainly affect one's health status in the long-run. Savings for retirement increase consumption in old-age but come at the cost of less consumption at present.

From a long-run perspective, people with a present-bias undervalue delayed benefits when deciding how much to consume and therefore act against their own long-run interests: asked in advance, they preferred a different consumption bundle than the one actually chosen, and would be better off if they could have committed themselves to

[1]For recent empirical evidence, see Ashraf et al. (2006), Brown et al. (2009), Dupas and Robinson (2013) or Augenblick et al. (2015).

do so. As a consequence, unregulated consumption allocations are inefficient.[2] People's current consumption imposes a positive "internal effect" on their long-run "selves", and they tend to under-consume goods that provide future benefits.[3] The presence of self-control problems might therefore justify corrective government interventions.

In this paper, we study the optimal form of such policies. Basically, governments employ two different types of policy instruments. First, most countries publicly provide goods like education, health care, or old-age consumption – typically at uniform levels and free of charge.[4] Second, private purchases of these goods are often subsidized by lowering market prices. In Germany, for example, up to 30 percent of tuition fees for private schools can be deducted from parents' taxable income. Likewise, many countries offer tax benefits for participants in private pension schemes. Further, it is common practice to implicitly subsidize pharmaceutical products by applying lower value-added tax rates (Pirttilä and Tuomala, 2004). Across countries, the usage of the two policy instruments differs: while price subsidies and public provision co-exist in some countries, others rely on only one instrument. E.g., Australia employs a public health care system, but citizens who buy private health insurance receive a rebate that covers part of the costs for their premiums. In contrast, in the United Kingdom, health care is exclusively publicly provided through the National Health service.

Both price subsidies and public provision can in principle correct inefficient consumer choices. While subsidies act through the price mechanism, public provision effectively imposes minimum quantities on each individual's consumption: by providing a private good and financing it through an equal-sized reduction in net incomes, governments can force individuals to consume more of goods that provide future benefits.[5]

An important question is then whether the two instruments are substitutes or comple-

[2]Time-inconsistent preferences raise the philosophical question of which preferences to use for welfare comparisons. In this paper, we follow related literature and apply a "long-run criterion" (Bernheim and Rangel, 2007). An allocation is called efficient if it maximizes people's long-run preferences. We therefore treat each individual's present-bias as an error (see Section 4.2 below).

[3]It is convenient to consider present-biased individuals as consisting of different selves (Laibson, 1997). In every period, choices are made by the respective current self. The term "internality" is used to distinguish the phenomenon from standard externality problems. With present-biased preferences, the external effect does not emerge across different individuals, but between different selves of the same individual (Herrnstein et al., 1993).

[4]In OECD countries, public spending on private goods and services ranges from 10 to 25 percent of GDP (Currie and Gahvari, 2008). By public provision we mean that the government *finances* a certain level of a private good. The actual provision may take place in public or private institutions.

[5]This argument requires that people cannot resell their provision levels, which is satisfied for goods like education or health care (Currie and Gahvari, 2008). Otherwise, public provision would be the same as a cash transfer and could not distort consumption choices.

ments in order to address people's self-control problems. Is one instrument superior compared to the other or should governments employ a combination of both? We consider a dynamic economy with two private goods. One good is standard, the other requires immediate expenses but yields utility in the future (e.g., health, education, etc.). The government can publicly provide or subsidize the good with delayed benefits. Subsidy rates and public provision levels must be identical for all individuals. Both policies are financed by taxes on exogenous gross incomes. People may differ in gross incomes and in the strength of their self-control problems, which we model as time-inconsistent, present-biased preferences (O'Donoghue and Rabin, 1999).

We find that optimal policies depend on the dimension of heterogeneity among individuals. In an economy with homogenous consumers, price subsidies and public provision are equivalent – in the sense that both can fully correct internal effects and implement an efficient allocation. With heterogeneous individuals, this equivalence disappears. In particular, when people only differ in their present-bias, public provision strictly dominates price subsidies. In this case, people are identical from a long-run perspective, and – as long as the government does not want to redistribute between identical individuals – the efficient consumption bundle is the same for all. However, due to differences in present-bias, people's short-run preferences (and thus, choices) differ. As a consequence, sustaining the efficient allocation through price subsidies would require type-specific subsidy rates – which is impossible when policies must be uniform. In contrast, by setting the provided amount to the efficient level, public provision can always restore efficiency.

When individuals have different biases and gross incomes, price subsidies and public provision can be complements. In the special case with only two income groups, this happens if and only if the poor have the stronger present bias. Moreover, optimal policies implement efficient consumption allocations. In the reverse case where the present-bias is more pronounced among the rich, price subsidies and public provision are substitutes: optimal policies include only one of the two instruments and are second-best in that efficient allocations cannot be achieved. In scenarios with more than two types, our results generalize if the strength of present-bias is monotonically related to individuals' gross incomes. I.e., governments should employ a combination of both instruments if self-control problems weaken with rising income. If the present-bias intensifies as individuals become richer, it is optimal to implement either a price subsidy or public provision. Optimal policies are, however, generally second-best: in the presence of more than two types of present-bias, one cannot correct all internal effects with only two uniform instruments.

The relevance of our analysis is supported by recent empirical evidence. For example, as argued by Oreopoulos (2007), present-biased preferences can help to explain why students in the U.S., Canada, or UK drop out of high school even though an additional year of schooling increases their lifetime spending by 15 percent. Likewise, a growing literature tries to understand why in developing countries investments in health, education, or physical capital – although offering high returns – remain relatively low (Banerjee and Duflo, 2007). One possible explanation is that people have self-control problems and understate the positive benefits of such investments. E.g., using data from a field experiment in Kenya, Dupas and Robinson (2013) find that an important fraction of their sample was present-biased and benefited from a commitment device forcing participants to increase investments in preventative health care.[6]

Our paper makes three contributions. First, we complement the discussion on whether governments should use price- or quantity-based instruments for policy interventions (see, for example, Guesnerie and Roberts, 1984; Blomquist and Christiansen, 1998; Boadway et al., 1998; Bovenberg and Goulder, 2002). Two recent studies address this question from a behavioral economics perspective. In Farhi and Gabaix (2015), quantity regulations may be superior to taxes when consumers misperceive prices. Heutel (2015) studies optimal policies for externality-generating durable goods and finds that energy efficiency standards may dominate fuel economy taxes if consumers are present-biased. As in our model, quantity instruments may complement or even outperform taxes and subsidies only if consumers are heterogeneous in the behavioral failure.

Second, we contribute to a growing literature on optimal public policies when people have present-biased preferences (Gruber and Köszegi, 2001; O'Donoghue and Rabin, 2003, 2006; Alcott et al.,2014; Guo and Krause, 2015). Previous work has shown that both price subsidies (Aronsson and Thunström, 2008) and public provision (Amador et al., 2006; Aronsson and Granlund, 2014) can mitigate the under-consumption of goods like health care or old-age consumption. However, as our results reveal, an analysis of optimal policies should consider both instruments simultaneously: when people have different present-biases, efficient allocations are typically not achievable by a single instrument, and a combination of price subsidies and public provision might be useful. In fact, as shown in the present paper, both instruments are complementary in

[6] Similar evidence is provided in Ashraf et al. (2006) or Duflo et al. (2011).

important cases.

Finally, our analysis is related to the "older" literature on paternalism and merit goods where for some reason, the government's and individuals' preferences differ (Sandmo, 1983; Besley, 1988; Blomquist and Micheletto, 2006; Pirttilä and Tenhunen, 2008). In our approach, however, although people's short-run tastes are disrespected, individuals would agree with the government's policies from a long-run perspective (Cremer and Pestieau, 2011). In fact, present-biased preferences provide a foundation for why people spend insufficiently on education or health care when left to their own devices. Closest to our paper is Munro (1992), who shows that public provision can complement taxes and subsidies in scenarios where governments and individuals hold different views about the probabilities of uncertain events. However, he does not characterize the optimal mix of public provision and price instruments.

The paper has the following structure. Section 4.2 introduces the theoretical framework, presents the basic problem, defines the set of government policies and provides a simple example. Section 4.3 studies optimal policies when people differ only in present-bias. A scenario with differences in present-bias and gross incomes is considered in Section 4.4. Section 4.5 generalizes our findings for a more general preference formulation. The final section concludes.

## 4.2 The model

### 4.2.1 Framework

**General:** Consider a dynamic economy in discrete time, indexed by $t = 1, ..., \mathcal{T}$. The economy is populated by a large number $N$ of individuals who can be of different types $i = 1, ..., I$. The number of each type is normalized to one, such that $I = N$. There are two private goods, denoted $c$ and $x$. In every period $t$, an individual of type $i$ has an endowment or gross income $y^i$ of good $c$. The second good, $x$, is produced in a competitive industry that uses a linear technology and $c$ as the only input. Hence, producer prices are fixed, and we normalize units such that prices equal one for both goods. Individuals cannot save and must divide their income in period $t$ on goods $c$ and $x$. Denote by $c_t^i$ and $x_t^i$ the consumption levels of type $i$ in period $t$, and a consumption allocation by $C_t := \left(c_t^1, x_t^1, ..., c_t^I, x_t^I\right)$. An allocation $C_t$ is called feasible if $\sum_{i=1}^{I} \left(c_t^i + x_t^i\right) \leq \sum_{i=1}^{I} y^i$.

**Preferences:** Individuals have preferences over the two private goods. Good $x$ has properties of an investment good in the sense that (part of) its benefits accrue in the future. This feature applies to goods like education, health care, old-age consumption, or (healthy) food. For simplicity, we assume that the consumption of $x_t$ in period $t$ solely yields utility in the subsequent period. Good $c$ is a standard consumption good and has no effect on future well-being. Specifically, in period $t$, individuals enjoy instantaneous utility

$$u_t(c_t, x_{t-1}) := v(c_t) + h(x_{t-1}). \tag{4.1}$$

Functions $v$ and $h$ are strictly increasing, twice continuously differentiable and strictly concave. We abbreviate by $v'(c_t) := dv(c_t)/dc_t$, $v''(c_t) := d^2v(c_t)/dc_t^2$, $h'(x_{t-1}) := dh(x_{t-1})/dx_{t-1}$ and $h''(x_{t-1}) := d^2h(x_{t-1})/dx_{t-1}^2$. In addition, we require $v'(c_t) \to \infty$ as $c_t \to 0$, $v'(c_t) \to 0$ as $c_t \to \infty$, $h'(x_t) \to \infty$ as $x_t \to 0$ and $h'(x_t) \to 0$ as $x_t \to \infty$. As a consequence, goods $c$ and $x$ are normal and individuals choose positive amounts of both when endowed with a positive level of income.

Beyond instantaneous utility, individuals care about future well-being. An individual's intertemporal preferences at time $t$ are represented by (see, e.g., Laibson, 1997)

$$U_t^i(u_t, ..., u_{\mathcal{T}}) := u_t + \beta^i \sum_{\tau=t+1}^{\mathcal{T}} u_\tau. \tag{4.2}$$

The parameter $0 < \beta^i \leq 1$ captures the idea that individuals may have a self-control problem: if $\beta^i < 1$, there is an "extra bias" in $t$ favoring $t$ over all future periods. As $t$ arrives, individual $i$ gives higher relative weight to $t$ than in every period before. Individuals may differ in the intensity or strength of their present-bias – reflected in the magnitude of $\beta^i$. We assume that $\beta^i < 1$ for at least one $i$.

Present-biased preferences represented by (4.2) are time-inconsistent. To see this, consider individual $i$'s preferences over consumption bundles $(c_t^i, x_t^i)$. In period $t$, the individual's marginal rate of substitution between $c_t^i$ and $x_t^i$ is given by $\beta^i h'(x_t^i)/v'(c_t^i)$, which differs from the marginal rate of substitution $h'(x_t^i)/v'(c_t^i)$ in any period prior to $t$. Hence, if $\beta^i < 1$, preferences in $t$ are not consistent with preferences in any previous period. By (4.1) and (4.2), the choice of $(c_t^i, x_t^i)$ is independent of all past and future decisions, i.e., individuals effectively maximize $\hat{u}^i(c_t^i, x_t^i) := v(c_t^i) + \beta^i h(x_t^i)$.

### 4.2.2 Efficient allocations

When individuals have different preferences at different points in time, any normative analysis faces the problem of which preferences should be used for welfare comparisons. In this paper we follow earlier literature and adopt a "long-run" criterion to evaluate consumption allocations (see, e.g., O'Donoghue and Rabin, 1999, 2003, 2006; Gruber and Köszegi, 2001). In particular, we assume that individuals have long-run preferences given by

$$U^*(u_t, ..., u_{\mathcal{T}}) := \sum_{\tau=t}^{\mathcal{T}} u_\tau, \tag{4.3}$$

and we compare any two allocations according to (4.3). From a long-run perspective, individuals would therefore maximize $u^*(c_t, x_t) := v(c_t) + h(x_t)$ in period $t$, and we treat their present-bias ($\beta^i < 1$) as an "error". As an alternative interpretation, one can think of every individual as consisting of different selves who are indexed by the period where they make a consumption choice (see, e.g., Laibson, 1997). E.g., a person's period-$t$-self chooses $(c_t^i, x_t^i)$ to maximize $U_t^i$. In every period, the preferences of an individual's current self then deviate from the preferences of her earlier selves. In the same way as a Samuelson-Bergson social welfare function aggregates preferences of different individuals, we can set up a welfare function that assigns different weights to the preferences of different selves of the same individual (Bernheim and Rangel, 2007). The long-run criterion is then a special case where the present-biased preferences of the current self receive zero weight.

In the following, we will call an allocation $C_t$ *superior* to an allocation $\tilde{C}_t$ if $u^*(c_t^i, x_t^i) \geq u^*(\tilde{c}_t^i, \tilde{x}_t^i)$ for all $i = 1, ..., I$ and $u^*(c_t^i, x_t^i) > u^*(\tilde{c}_t^i, \tilde{x}_t^i)$ for at least one $i$. We define an allocation as *efficient* if each individual allocates a given income according to $u^*(c, x)$, i.e., makes no errors when choosing a consumption bundle. Formally, any efficient allocation must satisfy

$$-v'(c_t^i) + h'(x_t^i) = 0 \quad \text{for all} \quad i = 1, ..., I. \tag{4.4}$$

We define the set of efficient allocations $\mathcal{E}$ as all feasible allocations $C_t$ that satisfy (4.4). An element in $\mathcal{E}$ is denoted by $C_t^*$.

### 4.2.3 Laissez-faire

Applying (4.3) as a welfare criterion implies a gap between individuals' unregulated short-run choices and their long-run preferences. To see this, consider the allocations $\hat{C}_t := (\hat{c}_t^1, \hat{x}_t^1, ..., \hat{c}_t^I, \hat{x}_t^I)$ in a *laissez-faire* without government intervention. In the laissez-faire, each individual chooses $(\hat{c}_t^i, \hat{x}_t^i)$ to maximize $v(c_t^i) + \beta^i h(x_t^i)$ subject to $c_t^i + x_t^i = y^i$. A comparison of the first-order condition

$$-v'(\hat{c}_t^i) + \beta^i h'(\hat{x}_t^i) = 0 \tag{4.5}$$

with (4.4) shows that, in every period, laissez-faire allocations are inefficient. Moreover, individuals with a present-bias under-consume [over-consume] good $x$ [$c$] – in the sense that a marginal increase in $x$ accompanied by a marginal decrease in $c$ increases their long-run utility. Intuitively, the consumption of good $x$ in period $t$ positively affects utility in the subsequent period. When individuals suffer from a present-bias, their period-$t$-selves – who actually decide how much to consume – only take a fraction $\beta$ of the benefits into account. As a consequence, $x_t$ imposes a positive internal effect on individuals' long-run selves. By implicit differentiation of (4.5), it follows that the under-consumption intensifies when people have a stronger present-bias, i.e., $\partial\hat{x}/\partial\beta > 0$.
The inefficiency of the laissez-faire allocations does not depend on the choice of the long-run criterion for welfare comparisons. In fact, $\hat{C}$ would be inefficient for any welfare function where long-run preferences (4.3) receive positive weight. As individuals have the same preferences and endowments in every period, they effectively face the same choice in all $t < \mathcal{T}$.[7] Unless stated otherwise, we will drop the time index in what follows.

### 4.2.4 Available policies and individual behavior

As laissez-faire allocations are inefficient, government interventions in markets for education, health care, or old-age consumption may be justified. Two policy instruments can be used to correct consumer behavior: price subsidies and public provision. Specifically, in every period $t$, the government may provide an amount $g$ of good $x$ free of charge. While individuals are allowed to purchase additional units on a market at

[7] In the last period $\mathcal{T}$, individuals make no forward-looking decisions and spend their entire net income on good $c$. As this choice is efficient, we henceforth neglect $\mathcal{T}$.

price $q$, $g$ cannot be resold (i.e., $x^i \geq g$).[8] In addition, a per-unit subsidy or tax $s$ on good $x$ can be employed, such that the consumer price is $q = 1 + s$. Due to informational asymmetries, implementation problems, arbitrage opportunities, or political constraints, subsidy rates and public provision levels must be identical across individuals. Both policies can be financed through income taxes $T^i = T(y^i)$. An individual's net income is then given by $b^i = y^i - T^i$. We will denote a policy by $P = (T^1, ..., T^I, s, g)$. The government announces the sequence of policies $(P_1, ..., P_T)$ before period 1 and can perfectly commit to stick with it.[9] Policies must be feasible and balance the government budget, i.e.,

$$\sum_{i=1}^{I}(T^i + sx^i) - Nqg \quad = 0. \tag{4.6}$$

For a given policy, an individual of type $i$ solves

$$\max_{c^i, x^i} v(c^i) + \beta^i h(x^i) \quad \text{s.t.} \quad c^i + q(x^i - g) \leq b \quad \text{and} \quad x^i \geq g. \tag{4.7}$$

By strict concavity of $v$ and $h$, (4.7) has a unique solution for every $P$, and there exist demand functions $c^i = c(b^i, q, g; \beta^i)$ and $x^i = x\,(b^i, q, g; \beta^i)$. Let

$$V^i(T^i, s, g; \beta^i) := u^*(c(b^i, q, g; \beta^i), x(b^i, q, g; \beta^i)) \tag{4.8}$$

be the indirect long-run utility of type $i$. To simplify the exposition, we express indirect utilities in terms of policy variables. We call a policy $P$ *superior* to a policy $\tilde{P}$ if $V^i(T^i, s, g; \beta^i) \geq V^i(\tilde{T}^i, \tilde{s}, \tilde{g}; \beta^i)$ for all $i = 1, ..., I$ and $V^i(T^i, s, g; \beta^i) > V^i(\tilde{T}^i, \tilde{s}, \tilde{g}; \beta^i)$ for at least one $i$. A policy $P$ is called *optimal* if there is no other feasible policy $\tilde{P}$ that is superior to $P$.

We can obtain the set of optimal policies by maximizing the indirect utility of type 1 given that (i) indirect utilities of the other types do not fall below levels $\bar{u}^i$ and (ii) the

[8] This may hold due to specific characteristics of the good or a legal ban. Without this assumption, any analysis of public provision becomes meaningless as $g$ were equivalent to a cash transfer.

[9] In some situations, it may be optimal for the government to deviate from the announced policy in some future period after individuals have revealed information about their self-control type. In this case, not only individuals but also governments would behave time-inconsistent. In the context of optimal non-linear income taxation, such problems have been recently analyzed by, e.g., Guo and Krause (2015). To make the analysis tractable we assume full commitment on part of the government and leave such extensions for future research.

government budget is balanced. Formally, any optimal policy solves

$$\max_{T^1,\dots,T^I,s,g} V^1(T^1,s,g;\beta^1) \qquad \text{s.t.} \tag{4.9}$$

$$\text{(i)} V^i(T^i,s,g;\beta^i) \geq \bar{u}^i, \quad \text{for} \quad i = 2,\dots,I$$
$$\text{(ii)} \sum_{i=1}^{I}(T^i + sx^i) - Nqg \geq 0.$$

We define $\mathcal{P}$ as the set of policies $P$ such that there exist levels $\bar{u}^i$ and the policy solves (4.9). We have to distinguish between optimal *policies* in $\mathcal{P}$ and efficient *allocations* in $\mathcal{E}$. In deriving $\mathcal{E}$, we assumed a social planner who had full discretion over the economy's resources: he could achieve all feasible allocations by directly assigning consumption bundles to different types of individuals. In contrast, with the restricted set of policies – uniform subsidies and public provision levels – governments cannot support every feasible allocation. As a consequence, optimal policies in $\mathcal{P}$ may be second-best in the sense that they cannot implement allocations in $\mathcal{E}$.

### 4.2.5 A simple example

To get an intuition why price subsidies and public provision are both valuable to correct the under-consumption of good $x$, consider a simple example with a single individual who has a present-bias. In this scenario, the unique efficient allocation $(c_*, x_*)$ maximizes $u^*(c,x)$ subject to $c + x = y$. In the next lemma, we show that each of the two policies – combined with appropriate income taxes – can support this allocation. While a subsidy corrects individuals' consumption choices through a change in relative prices, a public provision system effectively imposes a minimum consumption constraint.

**Lemma 4.1** *The efficient allocation $(c_*, x_*)$ can be implemented by either $P^s = (s_* x_*, s_*, 0)$ or $P^p = (g_*, 0, g_*)$, where $s_* = \beta - 1$ and $g_* = x_*$.*

**Proof:** The efficient allocation $(c_*, x_*)$ must satisfy

$$-v'(c_*) + h'(x_*) = 0. \tag{4.10}$$

When the individual chooses $(c_*, x_*)$, $P^s$ is feasible: inserting $P^s$ into (4.6) yields $c_* + x_* = y$. To see that policy $P^s$ supports $(c_*, x_*)$, note that the individual chooses $(c, x)$

according to

$$-qv'(c) + \beta h'(x) = 0. \tag{4.11}$$

Using $s_* = \beta - 1$ and $T_* = s_* x_*$, it follows that (4.11) coincides with (4.10). Hence, $P^s$ implements $(c_*, x_*)$.

A public provision system can induce an efficient allocation by setting $g = x_*$ and $T = x_*$. This requires that the individual takes $g$ and does not buy additional units of good $x$ on the market, i.e., that $c^i = b^i = y^i - T^i = c_*$ and $x^i = g = x_*$. An individual would buy more units of good $x$ if and only if

$$\beta \frac{h'(x_*)}{v'(c_*)} > 1. \tag{4.12}$$

As $h'(x_*)/v'(c_*) = 1$ by (4.10), this never holds when $\beta < 1$. Thus, $(c_*, x_*)$ can also be implemented through $P^p$. ■

Lemma 4.1 easily generalizes to the case of many homogeneous individuals. Price subsidies and public provision are then equally effective in order to correct internalities, as both could implement an efficient allocation. However, while illustrative, a scenario with identical individuals seems unrealistic. If fact, available empirical evidence suggests that the intensity of self-control problem varies across individuals and social groups (see, e.g., Paserman, 2008). In the following, we will therefore consider an economy where individuals are heterogeneous.

## 4.3 Differences in present-bias

We start with a scenario where individuals solely differ in the strength of their present-bias. To simplify the exposition, we consider a simple case with two different types $i = 1, 2$, where $\beta^1 < \beta^2 \leq 1$ and $y^1 = y^2 = y$.[10] Denote the average demand for good $x$ by $\bar{x} = (x^1 + x^2)/2$. We first show that price subsidies and public provision can achieve superior consumption allocations compared to the laissez-faire $\hat{C}$. We then study whether both instruments are substitutes or complements.

[10] All the results in this section easily extend to the case with a finite number of types $I$.

### 4.3.1 Price subsidies

Our first proposition gives a sufficient condition such that the introduction of a subsidy is superior to the *laissez-faire policy* $\hat{P} := (0, 0, 0, 0)$:

**Proposition 4.1** *There exists a policy* $P = (T^1, T^2, s, 0)$ *with* $s < 0$ *that is superior to* $\hat{P}$ *if at* $\hat{C}$*,*

$$(1-\beta^1)\beta^1 h'(\hat{x}^1) + (\bar{x} - \hat{x}^1)\left[v''(\hat{c}^1) + (\beta^1)^2 h''(\hat{x}^1)\right] > 0. \tag{4.13}$$

**Proof:** Starting from the laissez-faire policy $\hat{P}$, consider the introduction of a marginal subsidy on $x$, i.e., $ds < 0$ and small, accompanied by an increase in $T^1 = T^2 = T$ that balances the government budget (4.6). At $\hat{P}$, this is equivalent to

$$2dT + (\hat{x}^1 + \hat{x}^2)ds = 0 \quad \text{or} \quad dT = -\bar{x}ds, \tag{4.14}$$

To analyze how the marginal subsidy affects individuals' long-run utilities, totally differentiate $V^i$ with respect to $T$ and $s$. This yields

$$\begin{aligned} dV^i = &- \left[v'(\hat{c}^i) + (1-\beta^i)h'(\hat{x}^i)\frac{\partial x^i}{\partial b^i}\right] dT \\ &+ \left[-v'(\hat{c}^i)\hat{x}^i + (1-\beta^i)h'(\hat{x}^i)\frac{\partial x^i}{\partial q}\right] ds, \end{aligned} \tag{4.15}$$

Using that at $\hat{P}$, $dT = -\bar{x}ds$, $v'(\hat{c}^i) = \beta^i h'(\hat{x}^i)$, $q = 1$ and

$$\frac{\partial x^i}{\partial b^i} = \frac{v''(\hat{c}^i)}{v''(\hat{c}^i) + \beta^i h''(\hat{x}^i)} > 0, \qquad \frac{\partial x^i}{\partial q} = \frac{v'(\hat{c}^i) - x^i v''(\hat{c}^i)}{v''(\hat{c}^i) + \beta^i h''(\hat{x}^i)} < 0,$$

we get

$$dV^i = \frac{1}{A}\left[(1-\beta^i)\beta^i h'(\hat{x}^i) + (\bar{x} - \hat{x}^i)\left[v'(\hat{c}^i) + (\beta^i)^2 h''(\hat{x}^i)\right]\right] ds, \tag{4.16}$$

where $A := v''(\hat{c}^i) + \beta^i h''(\hat{x}^i)$. Since $ds < 0$ and $v$ and $h$ are strictly concave, $dV^i > 0$ is equivalent to

$$(1-\beta^i)\beta^i h'(\hat{x}^i) + (\bar{x} - \hat{x}^i)\left[v''(\hat{c}^i) + (\beta^i)^2 h''(\hat{x}^i)\right] > 0. \tag{4.17}$$

For individuals of type 2, (4.17) is positive since $\hat{x}^1 < \bar{x} < \hat{x}^2$ and $\beta^2 \leq 1$. Hence, they always benefit from the introduction of a marginal subsidy. For individuals of type 1, the first term in (4.17) is positive while the second is negative. Hence, $dV^1 > 0$ if and only if condition (4.13) holds. This proves the proposition. ■

To get an intuition for Proposition 4.1, consider the introduction of a marginal subsidy at $\hat{P}$ and note that it must be financed through income taxation. Such a policy has two effects on indirect long-run utilities. First, the subsidy lowers the relative price of good $x$ and induces individuals to consume more (in compensated terms) of it. As types with a present-bias under-consume $x$ in the laissez-faire, the change in the relative price *ceteris paribus* benefits them.[11] This "corrective" effect is represented by the first term in (4.13). Second, since the income tax payment is the same for both types, any uniform subsidy redistributes from individuals who consume more to those who consume less of good $x$. As the consumption of good $x$ increases with $\beta$, we have $\hat{x}^1 < \bar{x} < \hat{x}^2$ in the laissez-faire. As a consequence, individuals of type 2 unambiguously benefit from the introduction of a marginal subsidy. Type 1 is better off if the corrective effect outweighs the redistribution effect reflected in the second term in (4.13).
The question is when condition (4.13) is satisfied. *Ceteris paribus*, it holds when (i) the demand for good $x$ is sufficiently responsive to price changes and (ii) the consumption dispersion reflected by $(\bar{x} - \hat{x}^1)$ is small. The next example shows that (4.13) can indeed hold under reasonable assumptions on parameters.

**Example:** Assume that $v(c^i) = \ln c^i$ and $h(x^i) = \ln x^i$. Then, (4.13) reduces to $\beta^1\beta^2 < 1$, which always holds as $\beta^1 < \beta^2 \leq 1$. ■

### 4.3.2 Public provision

We next ask whether the publicly providing good $x$ can correct the under-consumption of good $x$. To study the efficiency effects of public provision, first consider the demand functions $c^i$ and $x^i$ obtained from problem (4.7). They have a piecewise form and can be expressed as follows:[12]

$$c^i = c(b^i, q, g; \beta^i) = \begin{cases} c_d(b^i + qg, q; \beta^i) & \text{if } g < x_d(b^i + qg, q; \beta^i), \\ b^i & \text{if } g \geq x_d(b^i + qg, q; \beta^i), \end{cases} \tag{4.18}$$

[11]When $\beta^2 = 1$, individuals of type 2 were induced to consume "too much" of good $x$. However, since they are consuming optimally at $s = 0$, this effect is of second-order and therefore negligible.

[12]Demand functions (4.18) and (4.19) are continuous and differentiable in $b^i$, $q$ and $g$ with the exception that they have a non-differentiability when $g = x_d(b^i + qg, q; \beta^i)$. However, at such points the one-sided partial derivatives exist, which will suffice for all our proofs below.

$$x^i = x(b^i, q, g; \beta^i) = \begin{cases} x_d(b^i + qg, q; \beta^i) & \text{if } g < x_d(b^i + qg, q; \beta^i), \\ g & \text{if } g \geq x_d(b^i + qg, q; \beta^i). \end{cases} \tag{4.19}$$

In (4.18) and (4.19), $c_d^i = c_d(I^i, q; \beta^i)$ and $x_d^i = x_d(I^i, q; \beta^i)$ represent individual $i$'s "ordinary demands" when disposable income is $I^i$ and there is no public provision. The term $x_d(b^i + qg, q; \beta^i)$ then gives the amount of good $x$ an individual with net income $b^i$ demands if it receives the value of public provision in cash. When $x_d^i > g$, public provision acts as an income or cash transfer: since individual $i$ demands more than $g$ when endowed with an income of $b^i + qg$, she tops up public provision and purchases additional units on the market. If $x_d^i < g$, however, the individual desires less than the publicly provided amount. Since reselling is not feasible, she takes $g$ and spends her entire net income on good $c$. In this case, public provision constrains consumption choices.

Any feasible provision level must be financed through income taxes. In the absence of price subsidies ($q = 1$) and with identical gross incomes, running a balanced budget requires $T = g$ or $b = y - g$. It follows that $x_d(b + g, q; \beta^i) = x_d(y, q; \beta^i) = \hat{x}^i$ is equal to individual $i$'s laissez-faire consumption. Hence, if $g < \hat{x}^i$, public provision has no effect on individual behavior and long-run utility; individuals reduce their private purchases of $x$ by the amount $g$, but their total consumption levels do not change. In contrast, by setting $g > \hat{x}^i$ the government can force individuals to consume more of good $x$ than in the laissez-faire. The next proposition shows that there always exists a policy with positive public provision that is superior to $\hat{P}$.

**Proposition 4.2** *Public provision of $g = \hat{x}^i + dg$ is superior to $\hat{P}$, where $dg > 0$ and small.*

**Proof:** Consider a change from $\hat{P}$ to $P^e = (T_e^1, T_e^2, 0, g_e)$ with $g_e = \hat{x}^1$ and $T_e^1 = T_e^2 = g_e$. By demand functions (4.18) and (4.19), we have

$$x^1 = g = \hat{x}^1, \quad c^1 = b = \hat{c}^1, \quad x^2 = x_d(y, 1; \beta^2) = \hat{x}^2, \quad \text{and} \quad c^2 = c_d(y, 1; \beta^2) = \hat{c}^2.$$

I.e., both types choose the same consumption bundle and obtain the same long-run utility as at $\hat{P}$. At $P^e$, individuals' indirect long-run utilities are

$$V^1(T^1, 0, g; \beta^1) = v(b) + h(g), \tag{4.20}$$

$$V^2(T^2, 0, g; \beta^2) = v(c_d(y, 1; \beta^2)) + h(x_d(y, 1; \beta^2)). \tag{4.21}$$

Inserting $P^e$ into the government budget constraint (4.6) proves that $P^e$ is feasible. Now, consider a marginal increase in $g$ such that $g = \hat{x}^1 + dg$, where $dg \to 0$. Feasibility then requires that $dT = dg$. By (4.20) and (4.21), the effect on long-run well being of both types is

$$dV^1 = (1 - \beta^1)h'(g)dg > 0, \tag{4.22}$$

$$dV^2 = 0, \tag{4.23}$$

where we used that $-v'(\hat{c}^i) + \beta^i h'(\hat{x}^i) = 0$ at $P^e$. Since individuals of type 1 are better off compared to $\hat{P}$, the policy with $g = \hat{x}^i + dg$ is superior to $\hat{P}$. This proves Proposition 4.2. ■

Intuitively, the government can force individuals of type 1 to consume more of good $x$ by setting the public provision level marginally above $\hat{x}^1$. While individuals of type 2 are not affected by this policy ($\hat{x}^1 < \hat{x}^2$), those of type 1 benefit from a long-run perspective. Such a policy satisfies what some behavioral economists call "minimal interventions": when governments want to help individuals overcome their self-control problems, policies should not hurt people with full self-control.[13] Here, a public provision level slightly above $\hat{x}^1$ has no impact on individuals with $\beta^i = 1$ as they had chosen this amount anyway.

### 4.3.3 Price subsidies vs. public provision

We now analyze optimal policies when the government can employ both price subsidies and public provision. Any optimal policy that solves (4.9) should ideally implement an efficient consumption allocation defined by (4.4). Individuals who differ only in $\beta$ are identical from a long-run perspective: they have the same preferences (4.3) and gross incomes $y$. If governments do not want to redistribute between identical individuals, there is a unique, symmetric efficient allocation $C_s^*$ where both types receive the same consumption bundle $(c_*, x_*)$ and long-run utility $\bar{u}^* := u^*(c_*, x_*)$. In the next proposition, we show that there exists exactly one policy $P$ that supports $C_s^*$. This policy includes public provision but no price subsidy. We have

**Proposition 4.3** *When individuals differ only in $\beta^i$, for all $\bar{u}^2 \leq \bar{u}^*$ the optimal policy*

[13] See, e.g., Camerer et al. (2003) or Sunstein and Thaler (2003).

*is* $P^* = (T^1_*, T^2_*, 0, g_*)$ *with* $T^1_* = T^2_* = g_*$ *and* $g_* = x_*$*. This policy implements the symmetric efficient allocation* $C^*_s$ *where* $(c_*, x_*)$ *is the same for both types.*

**Proof:** The proof proceeds as follows. We first show that $P^*$ implements $C^*_s$ and is feasible. We then proof that $P^*$ is the only policy that supports $(c_*, x_*)$. In a last step, we show that for all utility levels $\bar{u}^2 \leq \bar{u}^*$, $P^*$ solves problem (4.9).
Under $P^*$, both types choose $(c_*, x_*)$: as $\beta^2 \leq 1$, we have $g_* \geq x_d(y, 1, \beta^2)$. By (4.18) and (4.19) it follows that $x^2 = g^*$ and $c^2 = b^*$. Since $\partial x^i_d / \partial \beta^i > 0$, $x^1 = g^*$ and $c^1 = b^*$. Inserting $P^*$ into (4.6) implies that it is feasible.
To see that $P^*$ is the only policy that can implement $(c_*, x_*)$, note that any policy with $g > x_*$ clearly cannot induce efficiency. In addition, policies where $s < 0$ and $g = 0$ cannot support $(c_*, x_*)$ either: By (4.5), efficiency requires $-v'(c^i) + h'(x^i) = 0$. This necessitates a subsidy of $s = (\beta^i - 1)$ for type $i$. Since individuals differ in $\beta^i$, type-specific subsidies are needed. However, this is impossible when $s$ is uniform. Finally, any policy with $s < 0$ and $g < x_*$ can be ruled out: when $x^i_d > g$ for both types, at least one of them does not choose $(c_*, x_*)$ as $\partial x^i_d / \partial \beta^i > 0$. If $x^2_d > g > x^1_d$, type 1 consumes less than $x^*$. If $x_* > g > x^2_d > x^1_d$, the same holds for both types.
Finally, note that $(c_*, x_*)$ maximizes long-run utility $u^*(c, x)$ of type 1 when $\bar{u}^2 = \bar{u}^*$. As this allocation can be implemented if and only if $P = P^*$, $P^*$ solves (4.9) when $\bar{u}^2 = \bar{u}^*$. Further note that $V^1(T, s, g; \beta^1) \leq V^2(T, s, g; \beta^2)$ for every $P$. Hence, $P^*$ maximizes $V^1(T, s, g; \beta^1)$ also for $\bar{u}^2 < \bar{u}^*$. ■

To correct the inefficiencies from people's present-bias, governments should publicly provide goods like education or health care and not use price subsidies. The intuition is simple. When individuals differ in $\beta$ but have identical incomes, a price subsidy cannot implement the efficient allocation $C^*_s$: inducing both types to choose the bundle $(c_*, x_*)$ would require type-specific subsidy rates and income transfers – which is impossible since $s$ must be uniform. In contrast, with $g = x_*$ and net incomes adjusted accordingly, a public provision system can always support $(c_*, x_*)$ for both types. As a consequence, the policy $P^*$ is optimal if the government does not intend to redistribute between identical types 1 and 2, i.e., if $\bar{u}^2 \leq \bar{u}^*$.[14]

[14] When $\bar{u}^2 > \bar{u}^*$, any optimal policy that solves (4.9) includes a subsidy but no public provision and cannot implement an efficient allocation $C_* \in \mathcal{E}$: by setting $\bar{u}^2 > \bar{u}^*$, resources need to be redistributed from individuals of type 1 to those of type 2. As income transfers are infeasible for individuals with the same $y$, the only policy instrument that can achieve redistribution is the uniform subsidy (as $x^1_d < x^2_d$). However, as we abstract from redistributive motives and focus on policies that correct inefficiencies, this

We therefore have a situation where a quantity-based policy strictly dominates price instruments. A related result appears in Farhi and Gabaix (2015). They find that quantity regulations may outperform taxes when consumers misperceive prices. As in our model, the quantity instrument can be superior only if consumers are heterogeneous in the behavioral failure (in their case, inattention to prices). More generally, Guesnerie and Roberts (1984) showed that in second-best situations where Pareto efficient allocations are not attainable, quantity constraints like in-kind transfers or rationing may complement taxes and subsidies. In their model, inefficiencies originate from the presence of distortionary taxes. In the present analysis, consumer choices are inefficient in the absence of government intervention, but the inefficiency can be eliminated through public provision of good $x$.

## 4.4 Differences in present-bias and income

The set-up of the previous section is admittedly simple. In particular, individuals varied only in present-biases and were identical from a long-run perspective. This assumption might drive the superiority of public provision over price subsidies, as public provision can always implement the symmetric efficient allocation $(c^1_*, x^1_*) = (c^2_*, x^2_*)$. In this section, we allow individuals also to have different gross incomes. In general, efficient consumption bundles will then differ across types and a combination of policy instruments might be valuable. To keep the analysis tractable, we consider a finite number of types $i = 1, ..., I$ who differ in $y^i$ and (possibly) $\beta^i$, where $y^1 < y^2 < ... < y^I$. We assume that $x^1_d < ... < x^I_d$ always holds.[15] In addition, we restrict the set of efficient allocations $\mathcal{E}$ such that $c^1_* < ... < c^I_*$ and $x^1_* < ... < x^I_*$.[16]

### 4.4.1 Preliminaries

When individuals have different gross incomes, both price subsidies and public provision can always achieve superior allocations compared to policies $P = (T^1, ..., T^I, 0, 0)$ where

case is of minor interest. In fact, it is hard to justify why the government should intend to redistribute between identical individuals.

[15]Since both goods are normal and $\partial x^i_d/\partial\beta^i > 0$, this is always satisfied when $\beta^1 \leq ... \leq \beta^I$. However, as we will allow for $\beta^i > \beta^j$ for $i < j$, type $i$ might have a higher ordinary demand $x^i_d$ than type $j$. We henceforth exclude such cases.

[16]We make this assumption so that our distinction of the different types by their gross incomes remains meaningful. However, the assumption only serves to simplify the exposition and is not essential for any of our results.

governments only use income taxes:

**Lemma 4.2** *There always exist policies* $P^s = (T^1, ..., T^I, s, 0)$ *with* $s < 0$ *and* $P^p = (T^1, ..., T^I, 0, g)$ *with* $g > 0$ *that are superior to any policy where* $P = (T^1, ..., T^I, 0, 0)$.

**Proof:** The proof proceeds along the same lines as proofs of Propositions 4.2 and 4.1 and is therefore omitted. ■

Note that Lemma 4.2 includes the laissez-faire $\hat{P}$ as a special case. In contrast to the set-up of Section 4.3, a price subsidy is always useful as it does not redistribute within income classes.

In the following, we want to characterize optimal policies. We assume that subsidy rates and public provision levels cannot be made contingent on gross incomes $y^i$. Although this seems to be common in most countries, relying on anonymous policies requires some outside argument: as the government can observe gross incomes, it is not using the available information in a consistent manner (see, e.g., Anderberg, 2001). For example, administrative costs may prevent governments from implementing a complex system of income-dependent policies. Moreover, such policies might be infeasible due to equity or political economy considerations.[17]

It is instructive to start with the following question: is it possible to implement efficient allocations by using only one of the two instruments? Obviously, as we assumed $x_*^1 < ... < x_*^I$ and the provision level is uniform, the government cannot achieve allocations in $\mathcal{E}$ by a public provision system alone. However, the next lemma highlights a special case where uniform subsidies can support efficient allocations.

**Lemma 4.3** *Efficient allocations in* $\mathcal{E}$ *can be achieved through a policy* $P^s = (T^1, ..., T^I, s, 0)$ *with* $s = (\beta - 1)$ *if and only if* $\beta$ *is identical for all* $i = 1, ...I$.

**Proof:** Consider a given efficient allocation $C^* \in \mathcal{E}$. We show that this allocation can be implemented with policy $P^s = (T_s^1, ..., T_s^I, s_s, 0)$, where

$$T_s^i = y^i - c_*^i + q_s x_*^i, \quad s_s = \beta - 1 \quad \text{for} \quad i = 1, ..., I.$$

To proof that $P^s$ is feasible if each type chooses $(c_*^i, x_*^i)$, insert $P^s$ into (4.6). This yields

$$\sum_{i=1}^{I} (y^i - c_*^i - x_*^i) = 0, \tag{4.24}$$

[17] If one could condition subsidy rates and public provision levels on incomes, any efficient allocation $C_* \in \mathcal{E}$ can be achieved by either applying income-dependent subsidies or public provision levels.

which necessarily holds at any element in $\mathcal{E}^*$. Recall that individuals choose $c^i$ and $x^i$ such that

$$-qv'(c^i) + \beta^i h'(x^i) = 0 \quad \text{and} \quad b^i = c^i + qx^i. \tag{4.25}$$

Inserting $\beta^i = \beta$, $s_s = \beta - 1$ and $T_s^i$, we get

$$-v'(c^i) + h'(x^i) = 1 \quad \text{and} \quad c_*^i + q_* x_*^i = c^i + qx^i. \tag{4.26}$$

This holds if and only if $c^i = c_*^i$ and $x^i = x_*^i$. To prove the only if part, note that if $\beta^i \neq \beta^j$ for at least two $i \neq j$ and $s$ is uniform, for at least one type, we have

$$-v'(c^i) + h'(x^i) \neq 1, \tag{4.27}$$

and $C^*$ cannot be implemented. The same arguments apply for all $C_* \in \mathcal{E}$, which proves the lemma. ■

If all income groups have the same present-bias, each individual's consumption imposes the same marginal internality $\beta - 1$. Hence, a subsidy of $s = \beta - 1$ can induce all individuals to choose efficient consumption levels. This result is in line with findings from the literature on the taxation of consumption externalities. In particular, when all individuals exert the same marginal external effect, a Pigouvian tax on the externality-generating good can achieve Pareto efficient allocations (see, e.g., Myles, 1995) – provided that no other distortions exist. An important implication of Lemma 4.3 is that, as long as individuals differ in their present bias, public provision may be useful as an additional instrument. In the following, we characterize optimal policies when $\beta^i$ differs across income groups.

### 4.4.2 Two income types

To illustrate our main points, we start with the special case with only two types $i = 1, 2$ of gross incomes. As $y^1 < y^2$, we speak of type 1 as "the poor" and of type 2 as "the rich". We can state:

**Proposition 4.4** *(i) If $\beta^2 > \beta^1$, optimal policies in $\mathcal{P}$ are such that $g > 0$ and $s \leq 0$. Each $P \in \mathcal{P}$ can implement an efficient allocation $C^* \in \mathcal{E}$.*

*(ii) If $\beta^1 > \beta^2$, optimal policies in $\mathcal{P}$ are such that either $g > 0$ and $s = 0$ or $g = 0$*

*and $s < 0$. Optimal policies in $\mathcal{P}$ cannot implement efficient allocations $C^* \in \mathcal{E}$.*

**Proof:** See Appendix 4.A.1. ■

According to Proposition 4.4, whether price subsidies and public provision are complements or substitutes depends solely on the relative strength of both groups' present-bias. If the present-bias is stronger among the poor, an optimal policy typically consists of a combination of both instruments and implements an efficient allocation. Intuitively, the government can assign the efficient level $x^1_*$ to the poor through direct public provision; by setting $s = \beta^2 - 1$, it can induce the rich to choose $x^2_*$, which works since $x^1_* = g_* < x^2_*$. Therefore, both types' marginal internal effects can be separately targeted by using a price subsidy and public provision. An exception arises when the rich have no self-control problem, i.e., $\beta^2 = 1$. In this case, their consumption choices need not be "corrected", and the government can support any $C^* \in \mathcal{E}$ if and only if it exclusively employs public provision.

In the reverse case where the rich have the stronger self-control problem, optimal policies include either a price subsidy or public provision, but a combination is never optimal. This result is striking: although there are two sources of inefficiency and the government has two instruments at it's disposal, only one instrument is used in a policy optimum. The intuition is as follows. By the assumption $x^1_d < x^2_d$, the poor are constrained by lower levels of public provision. Hence, any policy mix must be such that $x^1_d < g < x^2_d$ and $s = (\beta^2 - 1)$; the consumption choice of the poor must be distorted through public provision, while choices of the rich are corrected by the subsidy.[18] When $\beta^1 > \beta^2$, however, such a policy is not incentive-compatible: the subsidy $s = (\beta^2 - 1)$ is so large (in absolute terms) that the poor would always top up $g$ to a larger level. In that case, a mix of public provision and subsidies cannot do better than a system of taxes and subsidies. Nevertheless, as each of the two instruments can attain a superior allocation compared to policies where $P = (T^1, T^2, 0, 0)$, optimal policies contain one of the two instruments. By the arguments made at the beginning of this section, such policies are second-best and cannot support efficient allocations. Which instrument is actually optimal depends on the form of individual preferences and the parameters of the model.

[18]If both types were constrained by public provision, i.e., $x^1_d < x^2_d < g$, any price subsidy would be redundant as individual demands are not responsive to price changes.

### 4.4.3 More than two income types

We now study the general case with a finite number of income types. As the policy space is restricted to uniform subsidy rates and provision levels, optimal policies are generally second-best: with more than two $\beta^i$'s, the government could restore efficiency only if it distorted individual's consumption behavior to different extents. However, this is impossible with only two policy instruments. [19]

As shown in the next proposition, the remaining results of Section 4.4 survive if $\beta^i$ is monotonic in gross incomes:

**Proposition 4.5** *(i) If $\beta^1 < ... < \beta^I$, optimal policies in $\mathcal{P}$ are such that $g > 0$ and $s \leq 0$.*

*(ii) If $\beta^1 > ... > \beta^I$, optimal policies in $\mathcal{P}$ are such that either $g > 0$ and $s = 0$ or $g = 0$ and $s < 0$.*

**Proof:** See Appendix 4.A.2. ■

When $\beta^i$ evolves non-monotonically across income groups, a characterization of optimal policies is more complicated. A sufficient condition for a policy mix to be optimal is $\beta^1 < \min\left\{\beta^2, ..., \beta^I\right\}$ and $\beta^I > \max\left\{\beta^1, ..., \beta^{I-1}\right\}$. However, as this condition is not necessary, price subsidies and public provision can be complementary also in scenarios where it is violated. Whether the two instruments are complements or substitutes therefore depends on the specific parameters of the model.

## 4.5 Non-separable preferences

So far, instantaneous preferences over goods $c$ and $x$ were additively separable. This assumption simplified the analysis considerably, as individuals' current behavior was independent of all past and future decisions. As a consequence, it is immaterial whether people know that their self-control will persist in the future or not (O'Donoghue and Rabin, 2006). When preferences are non-separable, however, the consumption choices of different selves of a single individual interact. For example, a person's choice of $(c_t^i, x_t^i)$ in period $t$ depends on (i) the level $x_{t-1}^i$ chosen in period $t-1$ and (ii) what the individual expects her future selves to select in all subsequent periods.

[19] The only exception is a special case where $\beta^1 < \beta^2 = ... = \beta^I$. Then, a combination of public provision and a price subsidy can support efficient allocations in $\mathcal{E}$. The proof of this result is identical to the proof of Proposition 4.4 and therefore omitted.

It then becomes important whether individuals are aware of their future present-bias or not. In general, there are two polar cases. If individuals are *naive*, they incorrectly believe that their self-control problem will disappear from the next period onwards, i.e., that their future selves will choose consumption bundles according to long-run preferences (4.3). On the other hand, individuals can be *sophisticated* and anticipate to be also present-biased in future periods. In this case, they know that future selves will maximize short-run preferences (4.2). As the behavior of future selves depends on current choices, a sophisticated individual might use current consumption strategically to manipulate the choices made by other selves in the future.[20,21] In this section, we analyze how optimal policies are affected if we drop the assumption of separable preferences.

### 4.5.1 Framework

Individuals' instantaneous preferences are represented by

$$u_t = u_t(c_t, x_{t-1}), \tag{4.28}$$

where $u$ is strictly increasing, twice continuously differentiable and strictly concave. In addition, we require $\partial u(c_t, x_{t-1})/\partial c_t \to \infty$ as $c_t \to 0$, $\partial u(c_t, x_{t-1})/\partial x_{t-1} \to \infty$ as $x_{t-1} \to 0$, $\partial u(c_t, x_{t-1})/\partial c_t \to 0$ as $c_t \to \infty$ and $\partial u(c_t, x_{t-1})/\partial x_{t-1} \to 0$ as $x_{t-1} \to \infty$. For simplicity, there are just three periods, i.e., $\mathcal{T} = 3$. Moreover, we consider a scenario with two types $i = 1, 2$ who differ in $\beta$ but have identical gross incomes. In the first period, individuals are endowed with an exogenous level $x_0$ of good $x$. Denote a consumption allocation in period $t$ by $C_t := (c_t^1, x_t^1, c_t^2, x_t^2)$ and the entire allocation by $C := (C_1, C_2, C_3)$.

The policy space is the same as in Section 4.2. Denote a policy in period $t$ by $P_t = (T_t^1, T_t^2, s_t, g_t)$ and a sequence of policies by $P = (P_1, P_2, P_3)$. The government announces the sequence of policies $P$ before period 1.

[20]Empirical evidence on the awareness of self-control problems is mixed (Fredrick et al., 2002). E.g., in experiments conducted by Hey and Lotito (2009), the majority of subjects were naive, while only a few were sophisticated. In contrast, Augenblick et al. (2015) report that 59 percent of the subjects in a real-effort experiment demanded a commitment-device, which suggests that many people know that they will have self-control problems in the future.

[21]As has been argued by O'Donoghue and Rabin (2001), there may also be intermediate types of sophistication. We leave such an extension for future research.

### 4.5.2 Individual behavior and laissez-faire

Since consumption choices are interdependent, each individual's behavior can be interpreted as the outcome of a sequential game between her different period-$t$-selves. Given a sequence of policies, we can characterize behavior by backward induction. Individual $i$'s period-3-self does not make any forward-looking decisions. Consequently, whatever happened earlier, it spends the entire net income on current consumption, i.e., $c_3^i = b_3^i$ and $x_3^i = 0$. In period 2, individual $i$ takes $x_1^i$ as fixed. Moreover, as it anticipates that $c_3^i = b_3^i$, it cannot use $x_2^i$ to influence choices in the next period. By our assumptions on preferences, there exist demand functions $c_2^i = c_2(b_2^i, q_2, g_2, b_3^i; x_1^i, \beta^i)$ and $x_2 = c_2^i(b_2^i, q_2, g_2, b_3^i; x_1^i, \beta^i)$.[22] In the first period, individual $i$ takes the demand functions of her future selves into account. Formally, $i$'s period-1-self solves

$$\max_{c_1^i, x_1^i} u_1(c_1^i, x_0) + \beta^i \left[u_2(c_2^i(b_2^i, q_2, g_2, b_3^i; x_1^i, \beta^i), x_1^i) + u_3(b_3^i, x_2^i(b_2^i, q_2, g_2, b_3^i; x_1^i, \beta^i))\right] \tag{4.29}$$

$$\text{s.t.} \quad b_1^i = c_1^i + q_1(x_1^i - g_1), \quad x_1^i \geq g_1.$$

To ensure a unique solution to (4.29), we assume that $U^1$ is strictly quasi-concave in $c_1^i$ and $x_1^i$.

In the next lemma, we show that the laissez-faire allocation $\hat{C} = (\hat{C}_1, \hat{C}_2, \hat{C}_3)$ is inefficient. In addition, while naive individuals always under-consume good $x$, those who are sophisticated may even *over-consume* good $x$ in the first period.

**Lemma 4.4** *If $\beta^i < 1$ for at least one type, $\hat{C}$ is inefficient. Moreover, individuals*

*(i) under-consume [over-consume] good $x$ [$c$] in period 2,*

*(ii) under-consume [over-consume] good $x$ [$c$] in period 1 if they are naive,*

*(iii) may over-consume [under-consume] good $x$ [$c$] in period 1 when they are sophisticated.*

**Proof:** See Appendix 4.A.3. ■

[22] In period 2 an individual of type $i$ maximizes

$$\max_{c_2^i, x_2^i} u_2(c_2^i, x_1^i) + \beta^i u_3(b_3^i, x_2^i) \quad \text{s.t.} \quad b_2^i = c_2^i + q_2(x_2^i - g_2), \quad x_2^i \geq g_2.$$

In period 2, individuals cannot influence the choices of their period-3-selves. Therefore, irrespective of whether they are naive or sophisticated, they behave qualitatively similar as in the case of non-separable preferences. In period 1, however, there may be incentives to influence one's future behavior through the choice of $x_1^i$. To see this, consider the first-order condition for problem (4.29) in the laissez-faire:

$$-\frac{\partial u_1(c_1^i, x_0)}{\partial c_1^i} + \beta^i \frac{\partial u_2(c_2^i, x_1^i)}{\partial x_1^i} + \beta^i \left[\frac{\partial u_2(c_2^i, x_1^i)}{\partial c_2^i} + \frac{\partial u_3(c_3^i, x_2^i)}{\partial x_2^i}\right] \frac{\partial x_2^i}{\partial x_1^i} = 0 \qquad (4.30)$$

The first two terms in (4.30) capture the intratemporal distortions due to self-control problems. Since the individual is present-biased, it's period-1-self takes only a fraction $\beta^i$ of the positive marginal benefits of $x_1^i$ into account, which is harmful from a long-run perspective. This effect was also present in previous sections and *ceteris paribus* calls for under-consumption of $x$. The incentive effects due to strategic interaction between individual $i$'s period-1- and period-2-selves are reflected in the third term of (4.30). For naive individuals, the term vanishes as they expect their future selves to choose $(c_2^i, x_2^i)$ according to $-\partial u_2\left(c_2^i, x_1^i\right)/\partial c_2^i + \partial u_3(c_3^i, x_2^i)/\partial x_2^i = 0$. When individuals are sophisticated, the bracketed term in (4.30) is positive. Hence, if $\partial x_2^i/\partial x_1^i < 0$, the presence of incentive effects strengthens the inefficiencies caused by present-biased preferences. However, if $\partial x_2^i/\partial x_1^i$ is positive and strong enough, sophisticated individuals may over-consume good $x$ in the first period. Formally, $\partial x_2^i/\partial x_1^i > 0$ can occur if $\partial^2 u_2/\partial c_2^i \partial x_1^i < 0$.

### 4.5.3 Optimal policy

If individuals under-consume good $x$ in both periods, optimal policies have the same properties as in Proposition 4.3. In this section, we therefore study the (more interesting) case where some people over-consume good $x$ in the laissez-faire. We abstract from redistributive motives and focus on the efficient allocation $C_s^*$ where both types receive the same consumption bundle $(c_t^*, x_t^*)$. For simplicity, only individuals of type 1 are present-biased (i.e., $\beta^2 = 1$). One might expect that the government needs to tax good $x$ in the first period to induce individuals of type 1 to lower $x_1^1$. However, as the next proposition shows, this intuition is incorrect:

**Proposition 4.6** *Assume that $\beta^1 < \beta^2 = 1$ and that individuals of type 1 over-consume good $x$ in the laissez-faire. Then, the optimal sequence of policies $(P_1, P_2, P_3)$ is such* *that* $P_1 = (T_1^1, T_1^2, 0, g_1)$*,* $P_2 = (T_2^1, T_2^2, 0, g_2)$ *and* $P_3 = (0, 0, 0, 0)$*, and implements an efficient allocation.*

**Proof:** See Appendix 4.A.4. ■

Intuitively, as soon as individuals are forced to consume the efficient consumption bundle $(c_2^*, x_2^*)$ in period 2, their period-1-selves have no incentive to use $x_1^i$ strategically. Public provision thus eliminates the incentives of each individual's period-1-self to self-commit in future periods. As a consequence, there is no over-consumption in period 1 and $(c_1^*, x_1^*)$ can be achieved by exclusively relying on public provision.

## 4.6 Conclusion

When people have present-biased preferences, they tend to under-consume goods that entail up front expenses and offer future benefits. We studied optimal public policies to correct inefficient consumption choices. In particular, we analyzed the relative merits of price subsidies for and public provision of goods like health care, education, or old-age consumption. We find that public provision strictly dominates price subsidies when people only differ in the severity of their self-control problem. In scenarios where people are also heterogeneous in gross incomes, it is crucial how the strength of individuals' present-bias evolves across income groups. If the present-bias intensifies as individuals become poorer, an optimal policy includes a combination of price subsidies and public provision. In the reverse case, it is optimal to use only one of both instruments.

Differences in the distribution of self-control problems might therefore provide a rationale for the different arrangements of public policies for education or health care across countries. However, empirical evidence on how the intensity of present-biases correlates with individuals' incomes or wealth is scarce. In experiments with Vietnamese Villagers, Tanaka et al. (2010) find no correlation between present-bias and wealth. In contrast, Paserman (2008) and Meier and Sprenger (2015) provide evidence suggesting that self-control problems are more pronounced among low- compared to high-income groups. As these results are inconclusive, further evidence is necessary to provide robust policy recommendations.

The set-up of our theoretical analysis is admittedly simple and could be extended in several directions. For example, we restricted available policies to price subsidies and public provision. However, when people differ along multiple dimensions, optimal policies identified in Section 4.4 are generally second-best and cannot implement efficient allocations. As a consequence, it might be useful to consider additional instruments like nudges in an analysis of optimal policies to correct internalities (see, e.g., Farhi and Gabaix, 2015).

Moreover, in Section 4.4 we abstracted from heterogeneity *within* income classes. If we allowed individuals with the same gross income to differ in present-bias, a characterization of optimal policies is more challenging.[23] The reason is that, within each income class, a uniform subsidy redistributes from individuals with a severe to those who have a relatively mild (or even no) present-bias. When such redistributive effects are strong, it may happen that both public provision and price subsidies are used in a policy optimum even if self-control problems intensify with rising income. We plan to study this extension in future work.

From a more general perspective, our results might also be relevant in other important contexts. For example, a growing literature deals with the implications of present-biased preferences for the consumption of "sin goods" like tobacco, alcohol, or unhealthy food. Unlike education or health care, these goods offer immediate benefits but may cause significant health costs in the future. People with a present-bias tend to over-consume sinful goods, and some government regulation is desirable. O'Donoghue and Rabin (2006) show that taxes on sin goods can improve people's long-run utilities – even for those who have no self-control problem. As the results of Sections 4.3 and 4.4 suggest, quantity restrictions might also be valuable for sin good regulation. In particular, when individuals differ in the intensity of self-control problems, maximum consumption constraints may complement or even outperform sin taxes.

A prominent example of a quantity restriction is the Sugary Drinks Portion Cap Rule, which prohibited the sale of many sweetened drinks exceeding a volume of 0,5 Liters in New York City. However, while specific taxes on tobacco or alcohol are indeed levied in many countries, quantity regulations are rare. Hence, depending on whether a good provides future benefits or future cost, we observe marked differences in the application of quantity restrictions in practice.[24] Whether these differences can be rationalized within a theoretical model is left as a question for future research.

[23]This, of course, only holds for uniform policies. When governments could condition policies on gross incomes, all results of Section 4.3 apply.

[24]One explanation might be that maximum quantities for certain goods are more difficult to enforce than minimum restrictions: compared to the provision of a minimum level of education or health care, governments need information about consumption levels of every single individual to prevent them from consuming more than certain amounts of specific goods such as tobacco or alcohol.

# Appendix 4.A

## 4.A.1 Proof of Proposition 4.4

Consider a given efficient allocation $C^* \in \mathcal{E}$ where $c_*^1 < c_*^2$ and $x_*^1 < x_*^2$.

**Item (i):** The proof proceeds as follows. We first show that if $\beta^2 > \beta^1$, $C^* = (c_*^1, x_*^1, c_*^2, x_*^2)$ can be supported by a policy $P^* = (T_*^1, T_*^2, s_*, g_*)$ with

$$g_* = x_*^1, \quad T_*^1 = y^1 - c_*^1, \quad T_*^2 = y^2 - c_*^2 - q_*(x_*^2 - g_*), \qquad (4.31)$$
$$s_* = \beta^2 - 1,$$

where $q_* = 1 + s_*$. We then argue that $P^*$ is feasible and there are no other policies that implement $C^*$. Hence, $P^*$ must be optimal.

First, note that, at $P^*$, $(c_*^1, x_*^1)$ is optimal for type 1 if $g_* > x_d(b_*^1 + q_* g_*, q_*; \beta^1)$, i.e., type 1 does not want to buy additional units of $x$ on the market (see demand functions (4.18) and (4.19)). This holds if and only if

$$\beta^1 \frac{h'(x_*^1)}{v'(c_*^1)} < 1 + s_* = \beta^2. \qquad (4.32)$$

We know that $h'(x_*^1)/v'(c_*^1) = 1$, since $C^*$ is efficient (see (4.4)). Thus, (4.32) reduces to

$$\beta^2 > \beta^1. \qquad (4.33)$$

Hence, type 1 chooses $(c_*^1, x_*^1)$ at $P^*$. For type 2, we have $x_d(b_*^2 + q_* g_*, q_*; \beta^2) = x_d(c_*^2 + q_* x_*^2, q_*; \beta^2) = x_*^2$ and $c_d(b_*^2 + q_* g_*, q_*; \beta^2) = c_d(c_*^2 + q_* x_*^2, q_*; \beta^2) = c_*^2$. The reason is that $I_*^2 = c_*^2 + q_* x_*^2$ is the disposable income necessary to induce individuals of type 2 to choose $(c_*^2, x_*^2)$ on the market when there is no public provision and the subsidy is $s = \beta^2 - 1$. Since $x_*^2 > x_*^1 = g_*$ by assumption, (4.18) and (4.19) imply that $x^2 = x_d^2$ and $c^2 = c_d^2$. Hence, type 2 selects $(c_*^2, x_*^2)$ and $P^*$ supports $C^*$.

Plugging $P^*$ into (4.6), we get

$$y^1 - c_*^1 - x_*^1 + y^2 - c_*^2 - x_*^2 = 0. \qquad (4.34)$$

By monotonicity of preferences, (4.34) holds at any $C^*$, and policy $P^*$ is feasible.

Finally, no other policy can achieve $C^*$. If $g > x_*^1$, type 1 does not choose $(c_*^1, x_*^1)$. If

$g < x^1_*$, type 2 chooses $(c^2_*, x^2_*)$ only if $s = \beta^2 - 1$. In this case, however, type 1 does never choose $(c^1_*, x^1_*)$. If $g = x^1_*$, no other combination of subsidies and net incomes can induce type 2 to select $(c^2_*, x^2_*)$. This proves item (i).

**Item (ii):** We first show that a policy *mix* cannot be optimal when $\beta^1 > \beta^2$. To see this, note that any optimal policy $P^*$ solves (4.9). If the optimum $P^*$ were interior in the sense that $g > 0$ and $s < 0$, we must have $x^1_d < g_* < x^2_d$: if $x^1_d < x^2_d < g_*$, demand functions (4.18) and (4.19) imply that $c^i = b^i$ and $x^i = g$ for both types, and levying a subsidy is useless. If $g_* < x^1_d < x^2_d$, public provision is cash equivalent for both types, so one can achieve the same allocation through a system of income taxes and price subsidies alone.
If policy $P^*$ were such that $x^1_d < g_* < x^2_d$, it must satisfy the first-order conditions

$$\frac{\partial L}{\partial T^1} = -v'_1 + \lambda = 0, \tag{4.35}$$
$$\frac{\partial L}{\partial T^2} = -\mu\left[v'_2 + (1-\beta^2)h'_2\frac{\partial x^2}{\partial b^2}\right] - \lambda\left(-1 + s\frac{\partial x^2}{\partial b^2}\right) = 0, \tag{4.36}$$
$$\frac{\partial L}{\partial s} = \mu\left[-v'_2(x^2 - g) + (1-\beta^2)h'_2\frac{\partial x^2}{\partial q}\right] + \lambda\left(x^2 - g + s\frac{\partial x^2}{\partial q}\right) = 0, \tag{4.37}$$
$$\frac{\partial L}{\partial g} = h'_1 + \mu\left[qv'_2 + (1-\beta^2)h'_2\frac{\partial x^2}{\partial g}\right] + \lambda\left[-1 + \left(-q + s\frac{\partial x^2}{\partial g}\right)\right] = 0, \tag{4.38}$$

where we abbreviated $v'_i := v'(c^i)$ and $h'_i := h'(x^i)$ and used that $\partial x^1/\partial b^1 = \partial x^1/\partial q = 0$ and $\partial x^1/\partial g = 1$. Combining (4.35) and (4.36) with (4.38) and using that $q\partial c^2/\partial b^2 = \partial c^2/\partial g$ yields

$$\frac{\partial L}{\partial g} = h'_1 - v'_1 = 0. \tag{4.39}$$

Add and substract $\beta^1 h'_1$ and $sv'_1$ to get

$$\frac{\partial L}{\partial g} = -qv'_1 + \beta^1 h'_1 + (1-\beta^1)h'_1 - sv'_1 = 0. \tag{4.40}$$

Since $x^1_d < g$ by assumption, it follows from individuals' optimization problem (4.7) that $-qv'_1 + \beta^1 h'_1 < 0$. Hence, we must have

$$(1-\beta^1)h'_1 - sv'_1 > 0. \tag{4.41}$$

Multiplying (4.36) by $x^2$ and combining it with (4.37) implies $s = \beta^2 - 1$. From (4.39), we have $h'_1 = v'_1$. Substituting these two expressions in (4.41) we get

$$\beta^2 > \beta^1, \tag{4.42}$$

which contradicts $\beta^1 > \beta^2$. As a consequence, the optimal policy $P^*$ cannot have $g > 0$ and $s < 0$.

By Lemma 4.2, for every $\bar{u}^2$, a policy where $P = (T^1, T^2, 0, 0)$ cannot be optimal. Hence, either a subsidy or public provision is used in an optimum. The second part of item (ii) follows since one instrument alone cannot implement $C^*$ when $\beta^1 \neq \beta^2$ and $\beta^1 > \beta^2$. The same arguments apply for all efficient allocations $C^* \in \mathcal{E}$ where $c^1_* < c^2_*$ and $x^1_* < x^2_*$. This proves Proposition 4.4. ■

## 4.A.2 Proof of Proposition 4.5

To shorten the presentation, we first prove Proposition 4.5 for a special case with $I = 3$. We then argue that the logic of the proof extends to any finite number of types $I$.

### 4.A.2.1 Three types

Consider given utility levels $\bar{u}^2$ and $\bar{u}^3$. Individuals' demand functions are given by (4.18) and (4.19). Any optimal policy $P^*$ solves problem (4.9). Denote the Lagrangian by:

$$\begin{aligned} L = {} & V^1(T^1, s, g; \beta^1) + \mu^2 \left[ V^2(T^2, s, g; \beta^2) - \bar{u}^2 \right] \\ & + \mu^3 \left[ V^3(T^3, s, g; \beta^3) - \bar{u}^3 \right] + \lambda \left[ \sum_{i=1}^{3} (T^i + s x^i) - Nqg \right], \end{aligned} \tag{4.43}$$

where $\mu^2$ and $\mu^3$ are the Lagrange multipliers for the minimum utility constraints and $\lambda$ is the multiplier for the government budget constraint.

$P^*$ must satisfy the first-order conditions

$$\frac{\partial L}{\partial T^1} = -\left[v_1' + (1-\beta^1)h_1'\frac{\partial x^1}{\partial b^1}\right] - \lambda\left(-1 + s\frac{\partial x^1}{\partial b^1}\right) = 0, \tag{4.44}$$

$$\frac{\partial L}{\partial T^2} = -\mu^2\left[v_2' + (1-\beta^2)h_2'\frac{\partial x^2}{\partial b^2}\right] - \lambda\left(-1 + s\frac{\partial x^2}{\partial b^2}\right) = 0, \tag{4.45}$$

$$\frac{\partial L}{\partial T^3} = -\mu^3\left[v_3' + (1-\beta^3)h_3'\frac{\partial x^3}{\partial b^3}\right] - \lambda\left(-1 + s\frac{\partial x^3}{\partial b^3}\right) = 0, \tag{4.46}$$

$$\begin{aligned}\frac{\partial L}{\partial s} &= -v_1'(x^1 - g) + (1-\beta^1)h_1'\frac{\partial x^1}{\partial q} \\ &+ \mu^2\left[-v_2'(x^2 - g) + (1-\beta^2)h_2'\frac{\partial x^2}{\partial q}\right] \\ &+ \mu^3\left[-v_3'(x^3 - g) + (1-\beta^3)h_3'\frac{\partial x^3}{\partial q}\right] + \lambda\sum_{i=1}^{3}\left(x^i - g + s\frac{\partial x^i}{\partial q}\right) = 0,\end{aligned} \tag{4.47}$$

$$\begin{aligned}\frac{\partial L}{\partial g} &= qv_1' + (1-\beta^1)h_1'\frac{\partial x^1}{\partial g} + \left[-qv_1' + \beta^1 h_1'\right]\frac{\partial x^1}{\partial g} \\ &+ \mu^2\left[qv_2' + (1-\beta^2)h_2'\frac{\partial x^2}{\partial g} + \left[-qv_2' + \beta^2 h_2'\right]\frac{\partial x^2}{\partial g}\right] \\ &+ \mu^3\left[qv_3' + (1-\beta^3)h_3'\frac{\partial x^3}{\partial g} + \left[-qv_3' + \beta^3 h_3'\right]\frac{\partial x^3}{\partial g}\right] \\ &+ \lambda\sum_{i=1}^{3}\left(-q + s\frac{\partial x^i}{\partial g}\right) = 0.\end{aligned} \tag{4.48}$$

Denote, respectively, the optimal policies in the absence of public provision ($g = 0$) by $P^s = (T_s^1, T_s^2, T_s^3, s_s, 0)$ and in the absence of a price subsidy ($s = 0$) by $P^p = (T_p^1, T_p^2, T_p^3, 0, g_p)$.

**Proof of item (i):** To prove item (i), we show that if $\beta^1 < \beta^2 < \beta^3$, there always exists a policy $P = (T^1, T^2, T^3, s, g)$ with $g > 0$ [$s < 0$] that is superior to $P^s = (T_s^1, T_s^2, T_s^3, s_s, 0)$ [$P^p = (T_p^1, T_p^2, T_p^3, 0, g_p)$].

We start with a situation where only $s$ is used. Note that $P^s$ is obtained by setting $g = 0$ in (4.9). It must satisfy (4.44), (4.45), (4.46) and (4.47). By combining (4.44), (4.45), (4.46) with (4.47), we can derive an implicit expression for $s_s$:

$$s_s = -\frac{\left[(1-\beta^1)h_1'\frac{\partial \tilde{x}^1}{\partial q}\alpha^1 + (1-\beta^2)h_2'\frac{\partial \tilde{x}^2}{\partial q}\alpha^2 + (1-\beta^3)h_3'\frac{\partial \tilde{x}^3}{\partial q}\alpha^3\right]}{\left[h_1'\frac{\partial \tilde{x}^1}{\partial q}\alpha^1 + h_2'\frac{\partial \tilde{x}^2}{\partial q}\alpha^2 + h_3'\frac{\partial \tilde{x}^3}{\partial q}\alpha^3\right]}, \tag{4.49}$$

where

$$\alpha^1 := \left[v_2' + (1-\beta^2)h_2'\frac{\partial x^2}{\partial b^2}\right]\left[v_3' + (1-\beta^3)h_3'\frac{\partial x^3}{\partial b^3}\right] > 0, \tag{4.50}$$

$$\alpha^2 := \left[v_1' + (1-\beta^1)h_1'\frac{\partial x^1}{\partial b^1}\right]\left[v_3' + (1-\beta^3)h_3'\frac{\partial x^3}{\partial b^3}\right] > 0, \tag{4.51}$$

$$\alpha^3 := \left[v_1' + (1-\beta^1)h_1'\frac{\partial x^1}{\partial b^1}\right]\left[v_2' + (1-\beta^2)h_2'\frac{\partial x^2}{\partial b^2}\right] > 0, \tag{4.52}$$

and $\tilde{x}^{\,i}$ is the compensated demand for good $x$ with $\partial\tilde{x}^i/\partial q = \partial c^i/\partial q + x^i \partial x^i/\partial b^i < 0$. As $0 < \beta^i \leq 1$, $s_s \in (-1, 0)$. Denote an individual's demanded quantities at $P^s$ by $c_s^i = c\left(b_s^i, q_s, 0; \beta^i\right)$ and $x_s^i = x\left(b_s^i, q_s, 0; \beta^i\right)$.

Consider a change from $P^s$ to a policy $P^r = (T_r^1, T_r^2, T_r^3, s_r, g_r)$ with

$$\begin{aligned} &T_r^1 = T_s^1 + q_r g_r, \quad T_r^2 = T_s^2 + q_r g_r, \quad T_r^3 = T_s^3 + q_r g_r, \\ &s_r = s_s, \quad g_r = x_s^1, \end{aligned}$$

where $q_r = 1 + s_r$. At $P_r$, we have $x_d^i = x_d(b_r^i + q_r g_r, q_r; \beta^i) = x_d(b_s^i, q_s; \beta^i) = x_s^i$. As $g_r = x_s^1 = x_d^1$ and we assumed $x_d^1 < x_d^2 < x_d^3$, by demands functions (4.18) and (4.19) individuals choose

$$\begin{aligned} &c^1 = b_r^1, \quad x^1 = g_r, \quad c^2 = c_d^2 = c_s^2, \quad x^2 = x_d^2 = x_s^2, \\ &c^3 = c_d^3 = c_s^3 \quad \text{and} \quad x^3 = x_d^3 = x_s^3. \end{aligned} \tag{4.53}$$

Hence, at policy $P^r$, all types choose the same consumption bundle and obtain the same long-run indirect utility as at $P^s$.

Inserting $P^r$ into the feasibility constraint (4.6) gives

$$\sum_{i=1}^{3}\left(T_s^i + s_s x^i(b_s^i, q_s, 0; \beta^i)\right) = 0. \tag{4.54}$$

The equality follows as (4.54) must bind at any solution to problem (4.9). Thus, $P^r$ is feasible.

At $P^r$, individuals of type 1 are just constrained by public provision, while those of types 2 and 3 are not constrained. We can therefore express indirect long-run utilities by

$$V^1(T^1, s, g; \beta^1) = v(b) + h(g), \tag{4.55}$$

$$V^2(T^2, s, g; \beta^2) = v(c_d^2(b^2 + qg, q; \beta^2)) + h(x_d^2(b^2 + qg, q; \beta^2)), \tag{4.56}$$

$$V^3(T^3, s, g; \beta^3) = v(c_d^3(b^3 + qg, q; \beta^3)) + h(x_d^3(b^3 + qg, q; \beta^3)). \tag{4.57}$$

Now, consider a change from $P^r$ to a policy $P^{r'} = (T_r^1 + dT^1, T_r^2 + dT^2, T_r^3 + dT^3, s_r, g_r + dg)$, where $dg > 0$, $dg \to 0$, $dT^1 = dg$, $dT^2 = q_s dg$ and $dT^3 = q_s dg$. Using, demand functions (4.18) and (4.19), it can be shown that individuals of type 1 are constrained by public provision also at $P^{r'}$, while those of types 2 and 3 are not. Plugging policy $P^{r'}$ into the government budget constraint (4.6) implies that $P^{r'}$ is feasible.
The change in indirect long-run utilities induced by the change from policy $P^r$ to $P^{r'}$ can thus be obtained by totally differentiating (4.55), (4.56) and (4.57) with respect to $T^i$ and $g$ at $P^r$

$$dV^1 = [s_s v_1' + (1 - \beta 1) h_1'] \, dg, \tag{4.58}$$

$$dV^2 = 0, \tag{4.59}$$

$$dV^3 = 0, \tag{4.60}$$

where we use that $dT^i = -db^i$ and $-qv'(c_s^i) + \beta^i h'(x_s^i) = 0$ at $P^r$.
From (4.58), it follows that introducing a public provision system is superior to $P^s$ if

$$s_s > -(1 - \beta 1) h_1' / v_1' \tag{4.61}$$

Inserting (4.49) gives, after some manipulations

$$\begin{aligned} &(\beta^2 - \beta^1) \frac{\partial \tilde{x}^2}{\partial q} \left[ v_3' + (1 - \beta^3) h_3' \frac{\partial x^3}{\partial b^3} \right] \\ &+ (\beta^3 - \beta^1) \frac{\partial \tilde{x}^3}{\partial q} \left[ v_2' + (1 - \beta^2) h_2' \frac{\partial x^2}{\partial b^2} \right] < 0, \end{aligned} \tag{4.62}$$

which holds as $\partial \tilde{x}^i / \partial q < 0$ and $\beta^1 < \beta^2 < \beta^3$. Consequently, there always exists a policy $P = (T^1, T^2, T^3, s, g)$ with $g > 0$ that is superior to $P^s$.
Now, we turn to the reverse case where, initially, only public provision is in place ($s = 0$). We obtain policy $P^p$ by setting $s = 0$ in (4.9). $P^p$ satisfies (4.44), (4.45), (4.46) and

(4.48).

We show that a change to a policy with $s < 0$ can always be superior to $P^p$. Note that $P^p$ can be of three different types. In the first, only individuals of type 1 are constrained in their consumption choice, or $x_d^1 < g_p < x_d^2 < x_d^3$. As $x_d^1 < g$, by (4.18) and (4.19), we have $\partial x^1/\partial b^1 = 0$ and $\partial x^1/\partial q = 0$. Consider the introduction of a marginal subsidy at $P^p$, financed through a rise in income taxes. The effect of this policy change on type 1's long-run utility is $-\partial L/\partial s$. Differentiation of (4.43) with respect to $s$ and combination with (4.44), (4.45) and (4.46) gives

$$\frac{\partial L}{\partial s} = \mu^2(1-\beta^2)h_2'\frac{\partial \tilde{x}^2}{\partial q} + \mu^3(1-\beta^3)h_3'\frac{\partial \tilde{x}^3}{\partial q} < 0. \tag{4.63}$$

Hence, introducing a marginal subsidy is superior to $P^p$. The same argument applies for the second type of optimum, where $x_d^1 < x_d^2 < g_p < x_d^3$.

When the optimum is of the third type, we have $x_d^1 < x_d^2 < x_d^3 < g_p$. Since individuals' demands then are $x^i = g$ and $c^i = b^i$, introducing a marginal subsidy has no effect on individuals' long-run utilities. However, there always exists a superior policy that consists of a mix of $g$ and $s$. To see this, consider the efficient allocation $C^*$ where $u^*(c_*^2, x_*^2) = \bar{u}^2$ and $u^*(c_*^3, x_*^3) = \bar{u}^3$. Note that, by monotonicity of preferences, at any optimal policy, the utility restrictions $V^2(T_p^2, 0, g_p; \beta^2) = \bar{u}^2$ and $V^3(T_p^3, 0, g_p; \beta^3) = \bar{u}^3$ strictly bind. Moreover, as $x_*^1 < x_*^2 < x_*^3$ but all types consume the same $x^i = g$, the optimum is second-best and we have $V^1(T_p^1, 0, g_p; \beta^1) < u^*(c_*^1, x_*^1)$. Denote by $R^m$ the minimal resources needed to attain utility levels $\bar{u}^2$ and $\bar{u}^3$. Since policy $P^p$ is second-best, the resources necessary to achieve $\bar{u}^2$ and $\bar{u}^3$, denoted $R^p$, exceed $R^m$. We now show that there always exists a policy that satisfies the utility restrictions with lower resources than $R^p$. As a consequence, the resources for type 1 are higher than at $P^p$, and an increase in type 1's long-run utility is possible.

We have to consider two different subcases. In the first, the optimum is such that $g_p > x_*^2$. Consider a switch from $P^p$ to $P^n$ with

$$\begin{aligned} &g_n = x_*^2, \quad T_n^2 = y^2 - c_*^2, \quad s_n = (\beta^3 - 1), \\ &T_n^3 = y^3 - c_*^3 - (1+s_n)(x_*^3 - g_n). \end{aligned} \tag{4.64}$$

The income tax of type 1 is adjusted such that the government budget constraint (4.6) holds. At $P^n$, type 3 chooses the bundle $(c_*^3, x_*^3)$. Type 2 selects $(c_*^2, x_*^2)$ as

$$\beta^2 \frac{h'(x_*^2)}{v'(c_*^2)} < 1 + s_n \quad \Longleftrightarrow \quad \beta^3 > \beta^2. \tag{4.65}$$

At $P^n$ the resources to attain $\bar{u}^2$ and $\bar{u}^3$ are equal to $R_m$ and thus, minimal. Consequently, the resources left for type 1 are higher than at $P^p$. Since $g_n = x_*^2 > x_*^1$ and by monotonicity and strict convexity of preferences, type 1 benefits from the proposed policy change.
In the second case, we have $g_p < x_*^2$. Then, we can introduce a policy $P^n$ with

$$g_n = g_p, \quad T_n^2 = T_p^2, \quad s_n = (\beta^3 - 1), \quad T_n^3 = y^3 - c_*^3 - (1 + s_n)(x_*^3 - g_n).$$

As individuals of type 3 choose $(c_*^3, x_*^3)$ at $P^n$, the resources to attain $\bar{u}^2$ and $\bar{u}^3$ are again lower than at $P^p$. Hence, the income tax of individuals of type 1 can be reduced, which benefits them as $g$ remains constant. Hence, $P^n$ is superior to $P^p$. This proves item (i) of Proposition 4.5.

**Proof of item (ii):** We show that if $\beta^1 > \beta^2 > \beta^3$, an optimal policy $P^*$ cannot be such that $x_d^1 < g_* < x_d^2 < x_d^3$ and $s_* < 0$ or $x_d^1 < x_d^2 < g_* < x_d^3$ and $s_* < 0$. To see this, assume the contrary. $P^*$ must satisfy (4.44), (4.45), (4.46), (4.47) and (4.48).
If $x_d^1 < g_* < x_d^2 < x_d^3$, by (4.18) and (4.19), the optimal subsidy satisfies

$$s_* = -\frac{\left[(1-\beta^2) h_2' \frac{\partial \tilde{x}^2}{\partial q} \alpha^2 + (1-\beta^3) h_3' \frac{\partial \tilde{x}^3}{\partial q} \alpha^3\right]}{\left[h_2' \frac{\partial \tilde{x}^2}{\partial q} \alpha^2 + h_3' \frac{\partial \tilde{x}^3}{\partial q} \alpha^3\right]}. \tag{4.66}$$

Combining (4.44), (4.45), (4.46) with (4.48) gives

$$\frac{\partial L}{\partial g} = h_1' - q v_1' + \lambda s = 0. \tag{4.67}$$

From (4.44), $\lambda = v_1'$. Using this and adding and substracting $\beta^1 h_1'$ we get

$$\frac{\partial L}{\partial g} = -q v_1' + h_1' + (1-\beta^1) h_1' + s v_1' = 0. \tag{4.68}$$

As $x_d^1 < g$ we have $-q v_1' + h_1' < 0$. Hence, $(1-\beta^1) h_1' + s v_1' > 0$. Inserting (4.66) and rearranging gives

$$(\beta^2 - \beta^1)h_2' \frac{\partial \tilde{x}^2}{\partial q}\alpha^2 + (\beta^3 - \beta^1)h_3' \frac{\partial \tilde{x}^3}{\partial q}\alpha^3 < 0, \tag{4.69}$$

which cannot hold when $\beta^1 > \beta^2 > \beta^3$. Thus, we have a contradiction, and an optimum cannot be such that $x_d^1 < g_* < x_d^2 < x_d^3$. and $s_* < 0$.

If the optimum were such that $x_d^1 < x_d^2 < g_* < x_d^3$ and $s_* < 0$, combination of (4.44), (4.45), (4.46) with (4.47) would imply

$$s_* = (\beta^3 - 1). \tag{4.70}$$

Combining (4.44), (4.45), (4.46) with (4.48) yields

$$\frac{\partial L}{\partial g} = h_1' - qv_1' + \lambda s + \mu^2 \left[h_2' - qv_2'\right] + \lambda s = 0. \tag{4.71}$$

By (4.44) and (4.45) we get

$$\lambda = v_1' \quad \text{and} \quad \mu^2 = \frac{v_1'}{v_2'}. \tag{4.72}$$

Substituting the expressions for $\lambda$ and $\mu^2$ in (4.71) and rearranging gives

$$\underbrace{\left[\beta^1 \frac{h_1'}{v_1'} - q\right]}_{<0} + \underbrace{\left[\beta^2 \frac{h_2'}{v_2'} - q\right]}_{<0} + (1 - \beta^1)\frac{h_1'}{v_1'} + (1 - \beta^2)\frac{h_2'}{v_2'} + 2s = 0. \tag{4.73}$$

Since $x_d^1 < x_d^2 < g_*$, the first two terms in (4.73) are negative. Hence, if $P^*$ were interior, we should have

$$(1 - \beta^1)\frac{h_1'}{v_1'} + (1 - \beta^2)\frac{h_2'}{v_2'} + 2s > 0. \tag{4.74}$$

Inserting (4.70) in (4.74) yields

$$\underbrace{\left[\frac{h_1'}{v_1'} - 1\right] + \left[\frac{h_2'}{v_2'} - 1\right]}_{=0} - \beta^1 \frac{h_1'}{v_1'} - \beta^2 \frac{h_2'}{v_2'} + 2\beta^3 > 0. \tag{4.75}$$

The first two terms in (4.75) are zero by (4.71). Dividing (4.75) by $\beta^3$ gives

$$1 - \underbrace{\frac{\beta^1}{\beta^3}\frac{h_1'}{v_1'}}_{>1} + 1 - \underbrace{\frac{\beta^2}{\beta^3}\frac{h_2'}{v_2'}}_{>1} > 0, \tag{4.76}$$

which is a contradiction as $\beta^1 > \beta^2 > \beta^3$. Consequently, we cannot have an interior optimum were both $g_* > 0$ and $s_* < 0$.
By Lemma 4.2, for given $\bar{u}^2$ and $\bar{u}^3$, a policy where $P = (T^1, T^2, T^3, 0, 0)$ cannot be optimal. Hence, either a subsidy or public provision is used in an optimum. This completes the proof of item (ii).

#### 4.A.2.2 Finite number of types

Now consider the case of a finite number of types $i = 1, ..., I$. The proof follows the same logic as in the three-types case.

**Item (i):** To show that there exists a policy $P$ with $g > 0$ that is superior to policy $P^s$ where only a subsidy is employed, we can derive an expression similar to (4.62), which now involves $I - 1$ terms, which are all smaller than zero as $\beta^1 < \beta^i$ for all $i = 2, ..., I$. By the same arguments as before, introducing a marginal subsidy is superior to the initial public provision system $P^p$ if at least one income type is not constrained, i.e., $g_p < x_d^i$ for at least one $i$. When all types are constrained at the initial optimum, we can switch to a policy with a subsidy $(\beta^I - 1)$ such that the richest income type chooses the efficient consumption bundle $(c_*^I, x_*^I)$. This policy always requires less resources than the initial policy $P^p$, and individuals of type 1 can be made better off without harming the remaining $I - 1$ types.

**Item (ii):** The proof of item (ii) easily extends to the case of $I$ types: by combining the now $I + 2$ first-order conditions for each type of potential optimum, we reach a contradiction to the hypothesis that a combination of $s$ and $g$ is optimal. ■

### 4.A.3 Proof of Lemma 4.4

Using long-run preferences (4.3), we define an allocation as efficient if it satisfies

$$-\frac{\partial u_1(c_1^*, x_0)}{\partial c_1^i} + \frac{\partial u_2(c_2^*, x_1^*)}{\partial x_1^i} = 0 \tag{4.77}$$

$$-\frac{\partial u_2(c_2^*, x_1^*)}{\partial c_2^i} + \frac{\partial u_3(c_3^*, x_2^*)}{\partial x_2^i} = 0. \tag{4.78}$$

Denote the set of efficient allocations by $\mathcal{E}$ and an element in $\mathcal{E}$ by $C^*$. In the laissez-faire, individuals choose goods $c$ and $x$ such that

$$
\begin{aligned}
&-\frac{\partial u_1(c_1^i, x_0)}{\partial c_1^i} + \beta^i \frac{\partial u_2(c_2^i, x_1^i}{\partial x_1^i} \qquad (4.79)\\
&+\beta^i \left[ -\frac{\partial u_2(c_2^i, x_1^i)}{\partial c_2^i} + \frac{\partial u_3(c_3^i, x_2^i)}{\partial x_2^i} \right] \frac{\partial x_2^i}{\partial x_1^i} = 0 \\
&-\frac{\partial u_2(c_2^i, x_1^i}{\partial c_2^i} + \beta^i \frac{\partial u_3(c_3^i, x_2^i)}{\partial x_2^i} = 0. \qquad (4.80)
\end{aligned}
$$

Comparing (4.77) and (4.79) and (4.78) and (4.80), respectively, proves that $C^*$ and $\hat{C}$ do not coincide. Hence, the laissez-faire allocation $\hat{C}$ is inefficient.

Item (i) follows since, from a long-run perspective, individual $i$'s marginal rate of substitution between $x_2^i$ and $c_2^i$ is

$$\frac{\partial u_3(c_3^i, x_2^i)/\partial x_2^i}{\partial u_2(c_2^i, x_1^i)/\partial c_2^i}. \qquad (4.81)$$

According to (4.80), in the laissez-faire, we have

$$\beta^i \frac{\partial u_3(c_3^i, x_2^i)/\partial x_2^i}{\partial u_2(c_2^i, x_1^i)/\partial c_2^i} = 1 \quad \Longrightarrow \frac{\partial u_3(c_3^i, x_2^i)/\partial x_2^i}{\partial u_2(c_2^i, x_1^i)/\partial c_2^i} > 1 \quad \Longleftrightarrow \quad \beta^i < 1. \qquad (4.82)$$

Hence, the marginal rate of substitution exceeds the marginal rate of transformation (which is equal to one) and an increase in $x$ financed by a reduction in $c$ can increase individual $i$'s long-run utility.

To prove item (ii), an analogous argument applies. Note that naive individuals expect their period-2-self to choose $x_2^i$ and $c_2^i$ such that $-\partial u_2(c_2^i, x_1^i)/\partial c_2^i + \partial u_3(c_3^i, x_2^i)/\partial x_2^i = 0$. Hence, the bracketed term in (4.79) vanishes and we have

$$\beta^i \frac{\partial u_2(c_2^i, x_1^i)/\partial x_1^i}{\partial u_1(c_1^i, x_0)/\partial c_1^i} = 1 \quad \Longrightarrow \frac{\partial u_2(c_2^i, x_1^i)/\partial x_1^i}{\partial u_1(c_1^i, x_0)/\partial c_1^i} > 1 \quad \Longleftrightarrow \quad \beta^i < 1. \qquad (4.83)$$

When individual $i$ is sophisticated, from (4.79) and (4.80) we get

$$\beta^i \frac{\partial u_2(c_2^i, x_1^i)/\partial x_1^i}{\partial u_1(c_1^i, x_0)/\partial c_1^i} + \beta^i(1-\beta^i) \frac{\partial u_3\left(c_3^i, x_2^i\right)/\partial x_2^i}{\partial u_1(c_1^i, x_0)/\partial c_1^i} \frac{\partial x_2^i}{\partial x_1^i} = 1. \qquad (4.84)$$

This implies

$$\frac{\partial u_2(c_2^i, x_1^i)/\partial x_1^i}{\partial u_1(c_1^i, x_0)/\partial c_1^i} > 1 \quad \Longleftrightarrow \quad \frac{\partial x_2^i}{\partial x_1^i} < 0. \qquad (4.85)$$

Item (iii) then follows since, by (4.84) we may have

$$\frac{\partial u_2(c_2^i, x_1^i)/\partial x_1^i}{\partial u_1(c_1^i, x_0)/\partial c_1^i} < 1 \tag{4.86}$$

when $\partial x_2^i/\partial x_1^i > 0$, i.e., when $\partial^2 u_2/\partial c_2^i \partial x_1^i < 0$.
In that case, marginally reducing good $x$ would increase type $i$'s long-run utility. ■

### 4.A.4 Proof of Proposition 4.6

We show that the symmetric efficient allocation $C_s^* := (c_1^*, x_1^*, c_2^*, x_2^*, c_3^*, x_3^*)$ can be implemented if and only if $P = P^*$. Note that in the last period, it is optimal to have $c_3^* = y$. Hence, $P_3^* = (0, 0, 0, 0)$. In periods 1 and 2, consider the feasible policies $P_1^*$ and $P_2^*$, where

$$g_1^* = x_1^*, \quad T_1^1 = T_1^2 = T_1^* = g_1^*, \quad s_1^* = 0, \quad g_2 = x_2^*, \quad T_2^1 = T_2^2 = T_2^* = g_2^*, \quad s_2^* = 0.$$

In both periods, public provision levels equal the efficient values. However, individuals are allowed to buy additional units of good $x$ on the market. Hence, we have to check whether $x_t^i = g_t$ and $c_t^i = b_t^i$ is indeed their optimal choice. In period 2, an individual of type $i$ wants to top up the public provision level if

$$\beta^i \frac{\partial u_3(c_3^*, x_2^*)/\partial x_2^i}{\partial u_2(c_2^*, x_1^*)/\partial c_2^i} > 1, \tag{4.87}$$

However, as

$$\frac{\partial u_3(c_3^*, x_2^*)/\partial x_2^i}{\partial u_2(c_2^*, x_1^*)/\partial c_2^i} = 1 \quad \text{and} \quad \beta^i \leq 1 \tag{4.88}$$

(4.87) cannot hold. Hence, in period 2, individuals choose $x_2^i = x_2^*$ and $c_2^i = c_2^*$. In period 1, $x_1^i = x_1^*$ and $c_1^i = c_1^*$ is not optimal if

$$\beta^i \frac{\partial u_2(c_2^*, x_1^*)/\partial x_1^i}{\partial u_1(c_1^*, \bar{x}_0)/\partial c_1^i} + \beta^i \frac{\partial u_2(c_2^*, x_1^*)/\partial c_2^i}{\partial u_1(c_1^*, \bar{x}_0)/\partial c_1^i} \left[ -\frac{\partial u_3(c_3^*, x_2^*)/\partial x_2^i}{\partial u_2(c_2^*, x_1^*)/\partial c_2^i} + 1 \right] \frac{\partial x_2^i}{\partial x_1^i} > 1. \tag{4.89}$$

Since

$$\frac{\partial u_2(c_2^*, x_1^*)/\partial x_1^i}{\partial u_1(c_1^*, \bar{x}_0)/\partial c_1^i} = 1 \quad \text{and} \quad \frac{\partial u_3(c_3^*, x_2^*)/\partial x_2^i}{\partial u_2(c_2^*, x_1^*)/\partial c_2^i} = 1, \tag{4.90}$$

(4.89) simplifies to $\beta^i > 1$, which is impossible as $\beta^i \leq 1$. Thus, in both periods, individuals select $(c_t^*, x_t^*)$. Hence, the policy $(P_1^*, P_2^*, P_3^*)$ implements the Pareto efficient allocation. By the same arguments made in the analysis of separable preferences, no other policy supports this allocation. ■

# References


Aaron, H. J. and G. M. von Fürstenberg (1971). The inefficiency of transfers in-kind: the case of housing assistance. Western Economic Journal 9, 184-191.

Akerlof, G. A. and R. E. Kranton (2000). Economics and identity. Quarterly Journal of Economics 115, 715-753.

Allcott, H., Mullainathan, S. and D. Taubinsky (2014). Energy policy with externalities and internalities. Journal of Public Economics 112, 72-88.

Alpizar, F., Carlsson, F. and O. Johansson-Stenman (2005). How much do we care about absolute versus relative income and consumption? Journal of Economic Behavior and Organization 56, 405-421.

Amador, M., Werning, I. and G.- M. Angeletos (2006). Commitment vs. flexibility. Econometrica 74, 365-396.

Anderberg, D. (2001). Social insurance with in-kind provision of private goods. Scandinavian Journal of Economics 103, 41-61.

Aronsson, T. and D. Granlund (2014). Present-biased preferences and publicly provided private goods. FinanzArchiv 70, 169-199.

Aronsson, T. and O. Johansson-Stenman (2008). When the Joneses' consumption hurts: optimal public good provision and nonlinear income taxation. Journal of Public Economics 92, 986-997.

Aronsson, T. and O. Johansson-Stenman (2010). Positional concerns in an OLG model: optimal labor and capital income taxation. International Economic Review 51, 1071-1095.

Aronsson, T. and L. Thunström (2008). A note on optimal paternalism and health capital subsidies. Economics Letters 101, 241-242.

Arrow, K. J. and P. S. Dasgupta (2009). Conspicuous consumption, inconspicuous leisure. Economic Journal 119, 497-516.

Ashraf, N., Karlan, D. and W. Yin (2006). Tying Odysseus to the mast: evidence from a commitment savings product in the Philippines. Quarterly Journal of Economics 121, 635-672.

Augenblick, N., Niederle, M. and C. Sprenger (2015). Quarterly Journal of Economics 130, 1067-1115.

Bagwell, L. S. and B. D. Bernheim (1996). Veblen effects in a theory of conspicuous consumption. American Economic Review 86, 349-373.

Balcer, Y. (1980). Taxation of externalities: direct vs. indirect. Journal of Public Economics 13, 121-129.

Banerjee, A. V. and E. Duflo (2007). The economic lives of the poor. Journal of Economic Perspectives 21, 141-168.

Barbera, S. and B. Moreno (2011). Top monotonicity for single peakedness, single crossing and the median voter result. Games and Economic Behavior 73, 345-359.

Barr, N. (2012). The economics of the welfare state. Oxford: Oxford University Press.

Barten, A. P. and V. Böhm (1982). Consumer theory. In: Arrow, K. J. and M. D. Intriligator (eds.). Handbook of Mathematical Economics 2, 381-429.

Bearse, P., Glomm, G. and E. Janeba (2000). Why poor countries rely mostly on redistribution in-kind. Journal of Public Economics 75, 463-481.

Bénabou, R. and J. Tirole (2011). Laws and norms. NBER Working Papers 17579.

Bernheim, B. D. and A. Rangel (2007). Behavioral public economics: welfare and policy analysis with non-standard decision makers. In: Diamond, P. and H. Vartiainen (eds.). Economic Institutions and Behavioral Economics. Princeton: University Press, 7-84.

Besley, T. (1988). A simple model for merit good arguments. Journal of Public Economics 35, 371-383.

Besley, T. and S. Coate (1991). Public provision of private goods and the redistribution of income. American Economic Review 81, 979-984.

Besley, T. and S. Coate (1992). Understanding welfare stigma: taxpayer resentment and statistical discrimination. Journal of Public Economics 48, 165-183.

Bilancini, E. and L. Boncinelli (2010). Preferences and normal goods: an easy-to-check necessary and sufficient condition. Economics Letters 108, 13-15.

Bilancini, E. and L. Boncinelli (2012). Redistribution and the notion of social status. Journal of Public Economics 96, 651-657.

Blackorby, C. and D. Donaldson (1988). Cash versus kind, self-selection, and efficient transfers. American Economic Review 78, 691-700.

Blomquist, S. and V. Christiansen (1998). Price subsidies versus public provision. International Tax and Public Finance 5, 283-306.

Blomquist, S. (1993). Interdependent behavior and the effect of taxes. Journal of Public Economics 51, 211-218.

Blomquist, S. and L. Micheletto (2006). Optimal redistributive taxation when government's and agents' preferences differ. Journal of Public Economics 90, 1215-1233.

Boadway, R., Marchand, M. and M. Sato (1998). Subsidies versus public provision of private goods as instruments for redistribution. Scandinavian Journal of Economics 100, 545-564.

Boskin, M. J. and E. Sheshinski (1978). Individual welfare depends upon relative income. Quarterly Journal of Economics 92, 589-601.

Bovenberg, A.L. and L. H. Goulder (2002). Environmental taxation and regulation. In: Auerbach, A. J. and M. Feldstein (eds.). Handbook of Public Economics 3, 1474-1545.

Bowles, S. and Y. Park (2005). Emulation, inequality, and work hours: was Thorstein Veblen right? Economic Journal 115, 397-412.

Brown, A., Chua, Z. E. and C. F. Camerer (2009). Learning and visceral temptation in dynamic savings experiments. Quarterly Journal of Economics 124, 197-231.

Brown, P. H., Bulte, E. and X. Zhang (2011). Positional spending and status seeking in rural China. Journal of Development Economics 96, 139-149.

Brunello, G. and L. Rocco (2008). Educational standards in private and public schools. Economic Journal 118, 1866-1887.

Camerer, C., Issacharoff, S., Loewenstein, G., O'Donoghue, T. and M. Rabin (2003). Regulation for conservatives: behavioral economics and the case for 'asymmetric paternalism'. University of Pennsylvania Law Review 151, 1211-1254.

Carlsson, F., Johansson-Stenman O. and P. Martinsson (2007). Do you enjoy having more than others? Survey evidence of positional goods. Economica 74, 586-598.

Case, A., Garrib, A., Menendez, A. and Olgiati, A. (2008). Paying the piper: the high cost of funerals in South Africa. NBER Working Paper No. 14456.

Charles, K. K., Hurst, E. and N. Roussanov (2009). Conspicuous consumption and race. Quarterly Journal of Economics 124, 425-467.

Clark, A. E., Frijters, P. and M. Shields (2008). Relative income, happiness, and utility: an explanation for the Easterlin paradox and other puzzles. Journal of Economic Literature 46, 95-144.

Clark, A. E. and A. J. Oswald (1998). Comparison-concave utility and following behavior in social and economic settings. Journal of Public Economics 70, 133-155.

Clark, A. E. and C. Senik (2010). Who compares to whom? The anatomy of income comparisons in Europe. Economic Journal 120, 573-594.

Coate, S. (1995). Altruism, the samaritan's dilemma, and government transfer policy. American Economic Review 85, 46-57.

Corneo, G. and H. P. Grüner (2000). Social limits to redistribution. American Economic Review 90, 1491-1507.

Corneo, G. and O. Jeanne (1997). Conspicuous consumption, snobbism and conformism. Journal of Public Economics 66, 55-71.

Corrazini, L., Espositoc, L. and F. Espositoc (2012). Reign in hell or serve in heaven? A cross-country journey into the relative vs absolute perceptions of well-being. Journal of Economic Behavior and Organization 81, 715-730.

Currie, J. (2003). U.S. food and nutrition programs. In: Moffitt, R. (ed.). Means-tested transfer programs in the United States. Chicago: University of Chicago Press for NBER, pp. 199-290.

Currie, J. (2006). The take-up of social benefits. In: Auerbach, A. J., Card, D. and J. Quigley (eds.). Poverty, the distribution of income, and public policy. New York: Russell Sage, pp. 80-148.

Currie, J. and F. Gahvari (2008). Transfers in cash and in-kind: theory meets the data. Journal of Economic Literature 46, 333-383.

Cremer, H. and P. Pestieau (2011). Myopia, redistribution and pensions. European Economic Review 55, 165-175.

DellaVigna, S. (2009). Psychology and economics: evidence from the field. Journal of Economic Literature 47, 315-372.

Diamond, P. A. (1973). Consumption externalities and imperfect corrective pricing. Bell Journal of Economic and Management Science 4, 526-538.

Drechsel-Grau, M. and K. D. Schmid (2014). Consumption-savings decisions under upward-looking comparisons. Journal of Economic Behavior and Organization 106, 254-268.

Duflo, E., Kremer, M. and J. Robinson (2011). Nudging farmers to use fertilizer: theory and experimental evidence from Kenya. American Economic Review 101, 2350-2390.

Duesenberry, J. (1949). Income, saving, and the theory of consumer behavior. Cambridge: Harvard University Press.

Dufwenberg, M., Heidhues, P., Kirchsteiger, G., Riedel F. and J. Sobel (2011). Other-regarding preferences in general equilibrium. Review of Economic Studies 78, 613-639.

Dupas, P. and J. Robinson (2013). Why don't the poor save more? Evidence from health savings experiments. American Economic Review 103, 1138-1171.

Dupor, B. and W.-F. Liu (2003). Jealousy and equilibrium overconsumption. American Economic Review 93, 423-428.

Durlauf, S. N. and Y. M. Ioannides (2010). Socail interactions. Annual Review of Economics 2, 451-478.

Eckerstorfer, P. and R. Wendner (2013). Asymmetric and non-atmospheric consumption externalities, and efficient consumption taxation. Journal of Public Economics 106,42-56.

Epple, D. and R. E. Romano (1996a). Ends against the middle: determining public service provision when there are private alternatives. Journal of Public Economics 62, 297-325.

Epple, D. and R. E. Romano (1996b). Public provision of private goods. Journal of Politcal Economy 104, 57-84.

Falk, A. and M. Knell (2004). Choosing the Joneses: endogenous goals and reference standards. Scandinavian Journal of Economics 106, 417-435.

Farhi, E. and X. Gabaix (2015). Optimal taxation with behavioral agents. NBER Working Paper 21524.

Festinger, L. (1954). A theory of social comparison processes. Human Relations 7, 117-140.

Figlio, D. and J. Stone (1999). Are private schools really better? In: Polachek, S. W. (eds.). Research in Labor Economics 17, 115-140.

Frank, R. H. (1984). Interdependent preferences and the competitive wage structure. RAND Journal of Economics 15, 510-520.

Frank, R. H. (1985a). Choosing the right pond. Human behavior and the quest for status. Oxford: Oxford University Press.

Frank, R. H. (1985b). The demand for unobservable and other nonpositional goods. American Economic Review 75, 101-116.

Frank, R. H. and O. Heffetz (2011). Preferences for status: evidence and economic implications. In: Benhabib, J., Bisin, A. and M. O. Jackson (eds.). Handbook of Socio Economics 1, 69-91.

Frederick, S., Loewenstein, G. and T. O'Donoghue (2002). Time discounting and time preference: a critical review. Journal of Economic Literature 40, 351-401.

Frey, B. S. and A. Stutzer (2002). What can economists learn from happiness research? Journal of Economic Literature 40, 402-435.

Friedrichsen J. (2015). Signals sell: designing a product line when consumers have social image concerns. Unpublished working paper.

Friehe, T. and M. Mechtel (2014). Conspicuous consumption and political regimes: evidence from East and West Germany. European Economic Review 67, 62-81.

Gasparini, L. C. and S. M. Pinto (2006). Equality of opportunity and optimal cash and in-kind policies. Journal of Public Economics 90, 143-169.

Glazer, A. and K. A. Konrad (1996). A signaling explanation for charity. American Economic Review 86, 1019-1028.

Glomm, C. and B. Ravikumar (1998). Opting out of publicly provided services: a majority voting result. Social Choice and Welfare 15, 187-199.

Green, J. and E. Sheshinski (1976). Direct versus indirect remedies for externalities. Journal of Political Economy 84, 797-808.

Gruber, J. and B. Köszegi (2001). Is addiction 'rational'? Theory and evidence. Quarterly Journal of Economics 116, 1261-1303.

Guesnerie, R. and K. Roberts (1984). Effective policy tools and quantity controls. Econometrica 52, 59-86.

Guo, J.-T. and A. Krause (2015). Dynamic nonlinear income taxation with quasi-hyperbolic discounting and no commitment. Journal of Economic Behavior and Organization 109, 101-119.

Heffetz, O. (2011). A test of conspicuous consumption: visibility and income elasticities. Review of Economics and Statistics 93, 1101-1117.

Heffetz, O. (2012). Who sees what? Demographics and the visibility of consumer expenditures. Journal of Economic Psychology 33, 801-818.

Herrnstein, R., Loewenstein, G., Prelec, D. and W. Vaughan Jr. (1993). Utility maximization and melioration: internalities in individual choice. Journal of Behavioral Decision Making 6, 149-185.

Heutel, G. (2015). Optimal policy instruments for externality-producing durable goods under present bias. Journal of Environmental Economics and Management 72, 54-70.

Hey, J. D. and G. Lotito (2009). Naive, resolute or sophisticated? A study of dynamic decision making. Journal of Risk and Uncertainty 38, 1-25.

Hirsch, F. (1976). Social limits to growth. Cambridge: Harvard University Press.

Hopkins E. and T. Kornienko (2004). Running to keep in the same place: consumer choice as a game of status. American Economic Review 94, 1085-1107.

Ireland, N. J. (1994). On limiting the market for status signals. Journal of Public Economics 53, 91-110.

Ireland, N. J. (1998). Status-seeking, income taxation and efficiency. Journal of Public Economics 70, 99-113.

Ireland, N. J. (2001). Optimal income tax in the presence of status effects. Journal of Public Economics 81, 193-212.

Khamis, M., Prakash, N. and Z. Siddique (2012). Consumption and social identity: evidence from India. Journal of Economic Behavior and Organization 83, 353-371.

Kuhn, P., Kooreman, P., Soetevent, A. and A. Kapteyn (2011). The effects of lottery prizes on winners and their neighbors: evidence from the Dutch postcode lottery. American Economic Review 101, 2226-2247.

Kumagai, S. (1980). An implicit function theorem: comment. Journal of Optimization Theory and Applications 31, 285-288.

Laibson, D. (1997). Golden eggs and hyperbolic discounting. Quarterly Journal of Economics 112, 443-477.

Leibenstein, H. (1950). Bandwagon, snob, and veblen effects in the theory of consumers' demand. Quarterly Journal of Economics 64, 183-207.

Levy, G. (2005). The politics of public provision of education. Quarterly Journal of Economics 120, 1507-1534.

Levy, G. and R. Razin (2015). Preferences over equality in the presence of costly income sorting. American Economic Journal: Microeconomics 7, 308-337.

Lindbeck, A., Nyberg, S. and J. W. Weibull (1999). Social norms and economic incentives in the welfare state. Quarterly Journal of Economics 114, 1-35.

Litman, T. (2009). Mobility as a positional good. Implications for transport policy and planning. In: T. Conley and A. MacLaren (eds.). Car troubles. Ashgate: London, pp. 199-218.

Luttmer, E. F. P. (2005). Neighbors as negatives: relative earnings and well-being. Quarterly Journal of Economics 120, 963-1002.

Luelfesmann, C. and G. M. Myers (2011). Two-tier public provision: comparing public systems. Journal of Public Economics 95, 1263-1271.

Martinez-Mora, F. (2006). The existence of non-elite private schools. Journal of Public Economics 90, 1505-1518.

Meier, S. and C. D. Sprenger (2015). Temporal stability of time preferences. Review of Economics and Statistics 97, 273-286.

Micheletto, L. (2008). Redistribution and optimal mixed taxation in the presence of consumption externalities. Journal of Public Economics 92, 2262-2274.

Moav, O. and Z. Neeman (2010). Status and poverty. Journal of the European Economic Association 8, 413-420.

Moav, O. and Z. Neeman (2012). Savings rates and poverty: the role of conspicuous consumption and human capital. Economic Journal 122, 933-956.

Moffitt, R. (1983). An economic model of welfare stigma. American Economic Review 75, 1023-1035.

Mumtaz, Z., Levay, A., Bhatti, A. and S. Salway (2013). Social Science and Medicine 94, 98-105.

Munro, A. (1992). In-kind distribution, uncertainty and merit wants: a simple model. Public Finance Review 20, 175-194.

Myles, G. D. (1995). Public Economics. Cambridge: University Press.

Nichols, A. L. and R. J. Zeckhauser (1982). Targeting transfers through restrictions on recipients. American Economic Review 72, 372-377.

O'Donoghue, T. and M. Rabin (1999). Doing it now or later. American Economic Review 89, 103-124.

O'Donoghue, T. and M. Rabin (2001). Choice and procrastination. Quarterly Journal of Economics, 116, 121-160.

O'Donoghue, T. and M. Rabin (2003). Studying optimal paternalism, illustrated by a model of sin taxes. American Economic Review (Papers and Proceedings) 93, 186-191.

O'Donoghue, T. and M. Rabin (2006). Optimal sin taxes. Journal of Public Economics 90, 1825-1849.

Oreopoulos, P. (2007). Do dropouts drop out too soon? Wealth, health and happiness from compulsory schooling. Journal of Public Economics 91, 2213-2229.

Paserman, M. D. (2008). Job search and hyperbolic discounting: structural estimation and policy evaluation. Economic Journal 118, 1418-1452.

Pesendorfer, W. (1995). Design innovation and fashion cycles, American Economic Review 85, 771-792.

Pirttilä, J. and S. Tenhunen (2008). Pawns and queens revisited: public provision of private goods when individuals make mistakes. International Tax and Public Finance 15, 599-619.

Pirttilä, J. and M. Tuomala (2004). Poverty alleviation and tax policy. European Economic Review 48, 1075-1090.

Postlewaite, A. (1998). The social basis of interdependent preferences. European Economic Review 42, 779-800.

Rayo, L. (2013). Monopolistic signal provision. B.E. Journals of Theoretical Economics 13, 27-58.

Roth, C. (2014). Conspicuous consumption and peer effects among the poor: evidence from a field experiment. CSAE Working Paper WPS/2014-29.

Runciman, W. G. (1966). Relative deprivation and social justice. London: Routledge and Kegan Paul.

Sandmo, A. (1975). Optimal taxation in the presence of externalities. Scandinavian Journal of economics 77, 86-98.

Sandmo, A. (1983). Ex post welfare economics and the theory of merit goods. Economica 50, 19-33.

Solnick, S. J. and D. Hemenway (2005). Are positional concerns stronger in some domains than in others? American Economic Review 95, 147-151.

Steg, L. (2005). Car use: lust and must. Instrumental, symbolic and affective motives for car use. Transportation Research Part A 39, 147-162.

Simanis, G. (1970). Social security abroad. Private health insurance in West Germany and Great Britain. Social Security Bulletin, Oct. 1970, 39-42.

Stiglitz, J. E. (1974). The demand for education in public and private school systems. Journal of Public Economics 3, 349-385.

Suls, J. and T. A. Wills (1991). Social comparison: contemporary theory and research. Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Sunstein, C. and R. H. Thaler (2003). Libertarian paternalism. American Economic Review (Papers and Proceedings) 93, 175-179.

Tanaka, T., Camerer, C. F. and Q. Nguyen (2010). Risk and time preferences: linking experimental and household survey data from Vietnam. American Economic Review 100, 557-571.

Truyts, T. (2010). Social Status and economic theory. Journal of Economic Surveys 24, 137-169.

Truyts, T. (2012). Signaling and indirect taxation. Journal of Public Economics 96, 331-340.

Vikander, N. (2015). Advertising to status-conscious consumers. Unpublished Working Paper.

Veblen, T. (1899). The theory of the leisure class: an economic study of institutions (reprinted 1994). New York: Dover Publications.

Wood, J. V. and K. L. Taylor (1991). Serving self-relevant goals through social comparison. In: Suls, J. and T. A. Wills (eds.). Social comparison: contemporary theory and research. Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.