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Facts of life with γ_5

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Abstract

The increasing precision of many experiments in elementary particle physics leads to continuing interest in perturbative higher order calculations in the electroweak Standard Model or extensions of it. Such calculations are of increasing complexity because more loops and/or more legs are considered. Correspondingly efficient computational methods are mandatory for many calculations. One problem which affects the feasibility of higher order calculations is the problem with γ_5 in dimensional regularization. Since the subject thirty years after its invention is still controversial I advocate here some ideas which seem not to be common knowledge but might shed some new light on the problem. I present arguments in favor of utilizing an anticommuting γ_5 and a simple 4–dimensional treatment of the hard anomalies.

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1. Introduction

The electroweak Standard Model (SM) [1] has been extremely successful in the interpretation of LEP/SLC data and higher order effects typically amount to 10 σ deviations if not taken into account [2]. These precise predictions are only possible due to the renormalizability [3] of the SM and the by now very precise knowledge of the relevant input parameters. Last but not least the relevant coupling constants are small enough such that perturbation theory mostly works very well.

The formal proofs of renormalizability of the SM [4] often relied on the assumption that a gauge invariant regularization exists. The question whether such a regularization exists is non–trivial because of the chiral structure of the fermions involved. At present the only regularization, which makes elaborate computations of radiative corrections feasible, is the dimensional regularization (DR) scheme [5, 6] which is well-defined for field theories with vectorial gauge symmetries only. However, in theories exhibiting chiral fermions, like the electroweak SM, problems with the continuation of the Dirac matrix γ_5 to dimensions $D \neq 4$ remain open within this context and several modifications of the 't Hooft–Veltman DR have been proposed [7, 8, 9, 10, 11, 12, 13, 14, 15]. It turns out that starting from the standard SM-Lagrangian and using a γ_5, which does not anticommute with the other Dirac matrices γ^μ, leads to "spurious anomalies" which violate chiral symmetry and hence gauge invariance. These anomalies would spoil renormalizability if we would not get rid of them by imposing "by hand" the relevant Ward-Takahashi (WT) [16] and Slavnov-Taylor (ST) [17, 18] identities order by order in perturbation theory [13, 19, 20, 21, 22]. At first sight this might not look to be a serious problem, however, violating the symmetries of the SM makes practical calculations much more difficult and tedious than they are anyway.

The problems of course are related to the existence of the Adler–Bell–Jackiw (ABJ) anomaly [23], which must cancel in the SM in order not to spoil its renormalizability [24].

Surprisingly, the prescriptions proposed and/or used by many authors continue to be controversial [9, 11, 13, 15, 20, 25, 26, 27, 28, 29, 30, 31, 32], and hence it seems to be necessary to reconsider the problem once again. We shall emphasize, in particular, the advantage of working with chiral fields. The consequences of working as closely as possible with chiral fields, it seems to me, has not been stressed sufficiently in the literature so far.

As a matter of principle it is important to mention two other approaches which both work in $D = 4$ dimensions. i) In quantum field theories on the lattice a recent breakthrough was the discovery of exact chiral invariance on the lattice [33] which circumvents the Nielsen–Ninomiya no–go theorem [34]. A well defined regularization which preserves simultaneously chiral–and gauge–symmetries is thus known and could be applied to the SM. ii) The algebraic renormalization of the electroweak SM to all orders [35] within the Bogoliubov-Parasiuk-Hepp-Zimmermann (BPHZ) framework is a mathematically well defined scheme, which is much more involved because it breaks the symmetries at intermediate stages and hence leads to much longer expressions which are extremely tedious to handle in practice. In cases of doubt this is the only known scheme which is free of ambiguities and works directly in 4–dimensional continuum field theory.

For perturbative calculations in the continuum we have to stick as much as possible to the more practical route of dimensional regularization. In the following tensor quantities in $D = 4$

dimensions are supposed to be defined by interpolation of $D = 2n$ ($n \geq 2$, integer) dimensions to dimensions below $D = 4$. It is well known that the γ–algebra, the so called "naive dimensional regularization" (NDR) *

$$\{\gamma^\mu, \gamma^\nu\} = 2g_{\mu\nu} \cdot \mathbf{1} \ , \ \ g^\mu{}_\mu = D \ , \ \ \mathrm{AC}(\mu) \equiv \{\gamma^\mu, \gamma_5\} = 0 \tag{1}$$

for dimensions of space–time $D = 4 - 2\epsilon$, $\epsilon \neq 0$ is inconsistent with

$$\mathrm{Tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_5) \neq 0 \ . \tag{2}$$

The latter condition is often considered to be necessary, however, for an acceptable regularization since at $D = 4$ we must find

$$\mathrm{Tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_5) = 4\mathrm{i}\varepsilon^{\mu\nu\rho\sigma} \ . \tag{3}$$

Generally, for γ_5 odd traces one obtains trace conditions from the cyclic property of traces. They are not fulfilled automatically, as we shall see, and hence the algebra is ill-defined in general. Considering $\mathrm{Tr}(\prod\limits_{j=0}^{4} \gamma^{\mu_j} \gamma_5 \gamma^\alpha)$ cyclicity requires

$$\mathrm{Tr}(\prod_{j=0}^{4} \gamma^{\mu_j} \mathrm{AC}(\alpha)) - 2 \sum_{i=0}^{4} (-1)^i g^{\alpha\mu_i} \mathrm{Tr}(\prod_{j=0,\, j\neq i}^{4} \gamma^{\mu_j} \gamma_5) = 0 \ . \tag{4}$$

Contraction with the metric tensor $g_{\alpha\mu_0}$ yields

$$2\,(g^\alpha{}_\alpha - 4)\, \mathrm{Tr}(\prod_{j=1}^{4} \gamma^{\mu_j} \gamma_5) + \mathrm{Tr}(\prod_{j=1}^{4} \gamma^{\mu_j} \mathrm{AC}(\gamma)) = 0 \tag{5}$$

with $\mathrm{AC}(\gamma) \equiv \gamma_\alpha \mathrm{AC}(\alpha)$. Thus $g^\alpha{}_\alpha = D \neq 4$ together with (2) implies $\mathrm{AC}(\mu) \neq 0$. However, non-anti-commutativity of γ_5 is in conflict with the chiral structure and hence with gauge invariance of the SM, in general. It is the purpose of this note to study the possibility of restoring gauge invariance by employing chiral fields systematically.

2. Formally gauge invariant Feynman rules

Obviously only terms involving γ^μ in the standard SM Lagrangian can be affected by a non–anticommuting γ_5. As an example we consider the leptonic part, given by

$$\begin{aligned} \mathcal{L}_\ell &= \bar{\ell}_\mathrm{R} i\gamma^\mu \, (\partial_\mu + ig' B_\mu)\ell_\mathrm{R} + \bar{\nu}_{\ell\mathrm{R}} i\gamma^\mu \partial_\mu \nu_{\ell\mathrm{R}} \\ &+ \ \bar{L}_\ell i\gamma^\mu \, (\partial_\mu + i\frac{g'}{2} B_\mu - ig\frac{\tau_a}{2} W_{\mu a})\, L_\ell \end{aligned} \tag{6}$$

using standard notation. As usual the chiral fields

$$\ell_\mathrm{R} = \Pi_+ \ell \ , \ \ \nu_{\ell\mathrm{R}} = \Pi_+ \nu_\ell \ , \ \ L_\ell = \begin{pmatrix} \nu \\ \ell \end{pmatrix}_\mathrm{L} = \Pi_- \begin{pmatrix} \nu \\ \ell \end{pmatrix} \tag{7}$$

*I read it as "normal dimensional regularization"