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Models for Fare Planning in Public Transport

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Abstract

The optimization of fare systems in public transit allows to pursue objectives such as the maximization of demand, revenue, profit, or social welfare. We propose a non-linear optimization approach to fare planning that is based on a detailed discrete choice model of user behavior. The approach allows to analyze different fare structures, optimization objectives, and operational scenarios involving, e.g., subsidies. We use the resulting models to compute optimized fare systems for the city of Potsdam, Germany.

1 Introduction

Fares are a direct and flexible instrument to influence passenger behavior and cost recovery of a public transport system. Setting fares is therefore a fundamental problem for any mass transit company or authority. The importance of this task is further increased by technological progress such as the introduction of electronic ticketing systems, which offer opportunities to implement versatile fare structures in the future.

Public transport fares are well investigated in the economic literature. They are often studied from a macroscopic point of view in terms of elasticities, equilibrium conditions, and marginal cost analyses in order to derive qualitative insights, see, e.g., Samuelson [17], Oum, Waters, and Yong [15], Curtin [5], Goodwin [8], and Pedersen [16]. Glaister and Collings [7] and Nash [14] proposed to treat the setting of fares as an *optimization problem*, namely, to maximize objectives such as revenue, passenger miles, or social welfare subject to a budget constraint. They also considered a number of practical issues such as different public transport modes or peak and off-peak times, solve the first order conditions of their models numerically, and report on implementations of the results at London Transport. More details, in particular, aspects of the network structure, were added in the approaches of Kocur and Hendrickson [10] and De Borger, Mayeres, Proost, and Wouters [6], primarily to model costs in a more precise way.

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In the meantime, the theory of discrete choice had emerged as a viable approach to model the behavior of passengers of a public transport system, see Ben-Akiva and Lerman [3] and McFadden [13]. Empirical studies give evidence that travel choice is governed by a number of factors, most notably travel time, availability of a car and of discounted long term tickets, and fares, see Albers [1] and Vrtic and Axhausen [22]. Many of these factors depend on the network structure. It therefore makes sense to integrate detailed models of passenger behavior into a network fare optimization approach. Advanced bilevel logit models based on this idea have recently been formulated by Lam and Zhou [12]. In this paper we also integrate these aspects. Going one step further, we show how a number of objectives and constraints of practical relevance can be formulated and that the resulting models can be solved. We apply these methods to optimize fares for the city of Potsdam in Germany.

Our results show that the structure of the network does indeed influence the behavior of the passengers, i.e., we demonstrate a "network effect". For example, passengers with identical travel times make different choices according to the relative attractiveness of competition by the car.

Problems that are related to, but are different from, public transit fare optimization include toll optimization resp. road pricing, see, e.g., Labbé et al. [11], Verhoef [21], and van Dender [19], and tariff zone design, see Hamacher and Schöbel [9].

The article is organized as follows. Section 2 introduces the fare planning problem. We propose a discrete choice demand model in Section 2.1, and a family of five fare optimization models in Section 2.2. The models address the following objectives and constraints:

- MAX-R is a basic model; it assumes a fixed level of service and maximizes revenue.
- MAX-P includes costs that depend on line operation frequencies and subsidies; the objective is to maximize profit.
- MAX-D maximizes the demand, i.e., the number of public transport passengers, subject to budget and capacity constraints.
- MAX-B maximizes the user benefit subject to budget and capacity constraints.
- MAX-S maximizes social welfare subject to capacity constraints.

The models are calibrated in Section 2.3 and used to compute and analyze fare systems for the city of Potsdam in Section 3. Solving the models numerically, we show that different fare systems can be compared and evaluated in a quantitative way and that fare systems can be designed and optimized in order to achieve the goals specified above. As far as we know, an analysis at this level of detail has not been done before in the context of public transit fares.

2 Fare Planing

The fare planning problem involves a public transportation network, i.e., a directed graph $G = (V, E)$, where the nodes V represent stations and the arcs A connections that can be used for travel. There is a set $D \subseteq V \times V$ of *origin-destination pairs* (OD-pairs or traffic relations) between which passengers want to travel. We assume fixed passenger routes, i.e., for every OD-pair (s, t) there is a unique directed path P_{st} through the network that the passengers will take when using public transport. In our case, passengers use the time-minimal path.

Furthermore, we are given a finite set $\mathcal{C}$ of *travel choices*. The subset $\mathcal{C}' \subset \mathcal{C}$ represents the travel choices of public transport. Examples of travel choices that we have in mind are using single or monthly tickets, distance dependent tickets, the car, etc. Travel choices also include the number of trips during a time horizon, e.g., 30 trips during a month with a monthly ticket.

We consider n nonnegative fare variables $x_1, \ldots, x_n$, which we call *fares*. Examples of fares are: a price per kilometer of travel, a price for crossing a zone, etc. A *fare vector* is a vector $\boldsymbol{x} \in \mathbb{R}^n_+$ of such fares.

We also consider price functions $p^i_{st} : \mathbb{R}^n_+ \to \mathbb{R}_+$ and demand functions $d^i_{st} : \mathbb{R}^n_+ \to \mathbb{R}_+$ for each OD-pair $(s, t) \in D$ and each travel choice $i \in \mathcal{C}$. The *price functions* $p^i_{st}(\boldsymbol{x})$ determine the price for traveling with travel choice i from s to t depending on the fare vector $\boldsymbol{x}$. All p^i_{st} appearing in this paper are affine functions and hence differentiable. The price functions for travel choices not using public transport do not depend on fares and are therefore constant. The *demand functions* $d^i_{st}(\boldsymbol{x})$ measure the number of passengers that travel from s to t with travel choice i, depending on the fare vector $\boldsymbol{x}$. The total demand for OD-pair $(s, t) \in D$ with public transport is

$$d^{\mathcal{C}'}_{st}(\boldsymbol{x}) := \sum_{i \in \mathcal{C}'} d^i_{st}(\boldsymbol{x}).$$

Usually, $d^{\mathcal{C}'}_{st}(\boldsymbol{x})$ is *non-increasing*. For one specific travel choice i, however, the demand function d^i_{st} does not necessarily have this property, because of substitution effects between different travel choices. In our application, the demand functions are differentiable and, in particular, continuous.

The remainder of this section is organized as follows. In Subsections 2.1 we specify a demand function. Fare planning models based on these definitions are proposed in Subsection 2.2. The models are calibrated with respect to data for the city of Potsdam in Subsection 2.3.

2.1 Demand Functions

A key feature of our fare planning models are the demand functions d^i_{st}. Based on a discrete choice logit approach, see Ben-Akiva and Lerman [3], we model them as follows. Consider a time horizon T, and assume that a passenger traveling from s to t performs a random number of trips $X_{st} \in \mathbb{Z}_+$ during T, i.e., X_{st} is a discrete random variable. We assume an upper bound N on X_{st}. In our approach, we define travel choices as follows. Passengers choose among a set A of *travel alternatives* (e.g., monthly ticket, single ticket, and car); we assume passengers do not mix alternatives, i.e., the same travel alternative is chosen for all trips. Let $A' \subset A$ be the set of travel alternatives involving public transport. The travel choices are then $\mathcal{C} = A \times \{1, \ldots, N\}$, and the travel choices for public transport are $\mathcal{C}' = A' \times \{1, \ldots, N\}$. Associated with each choice $(a, k) \in \mathcal{C}$ is a utility

$$U_{st}^{a,k}(\boldsymbol{x}) = V_{st}^{a,k}(\boldsymbol{x}) + \nu_{st}^{a},$$

which depends on the fare vector $\boldsymbol{x}$. Here, $V_{st}^{a,k}$ is a deterministic or observable utility, and ν_{st}^{a} is a random utility or disturbance term, which is $G(\eta, \mu)$ Gumbel distributed with $\eta = 0$. In our models, the deterministic utility is measured in monetary units and always includes the price function for public transport, i.e.,

$$V_{st}^{a,k}(\boldsymbol{x}) = W_{st}^{a,k} - p_{st}^{a,k}(\boldsymbol{x}). \tag{1}$$

Here, $W_{st}^{a,k}$ is a constant that subsumes all deterministic utilities that do not depend on fares such as travel time.

To simplify notation, we write $d_{st}^{a,k}(\boldsymbol{x})$ for the number of passengers traveling k times during T with alternative a from s to t and similarly $p_{st}^{a,k}(\boldsymbol{x})$ for the price of these trips. In logit models one assumes that each passenger takes the alternative of maximal utility. Using standard logit techniques, see Ben-Akiva and Lerman [3], it follows that the *expected* demand can be computed via an explicit formula as

$$d_{st}^{a,k}(\boldsymbol{x}) \;=\; \rho_{st} \cdot \frac{e^{\mu V_{st}^{a,k}(\boldsymbol{x})}}{\sum\limits_{b \in A} e^{\mu V_{st}^{b,k}(\boldsymbol{x})}} \cdot \mathbb{P}[X_{st} = k],$$

where ρ_{st} is the total number of passengers traveling from s to t. The last term computes the probability that passengers from s to t make k trips, while the middle term corresponds to the probability that they use alternative a. The formula expresses the expected demand over the probability spaces for X_{st} and the disturbance terms ν_{st}^{a}.

Note that $d_{st}^{a,k}(\boldsymbol{x})$ is continuous and even differentiable if the deterministic utilities $V_{st}^{a,k}(\boldsymbol{x})$ have this property. This is, for instance, the case for affine functions, see Subsection 2.3 and Section 3.

2.2 Fare Planning Models

We now propose five fare planning models that capture different aspects and objectives, whose appropriateness depends on the respective planning goals.