



## Massive QCD amplitudes at higher orders

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We consider the factorization properties of on-shell QCD amplitudes with massive partons in the limit when all kinematical invariants are large compared to the parton mass and discuss the structure of their infrared singularities. The dimensionally regulated soft poles and the large collinear logarithms of the parton masses exponentiate to all orders. Based on this factorization a simple relation between massless and massive scattering amplitudes in gauge theories can be established. We present recent applications of this relation for the calculation of the two-loop virtual QCD corrections to the hadro-production of heavy quarks.

*“Matter To The Deepest”, September 5-11 , 2007, Ustron, Poland.  
8th International Symposium on Radiative Corrections (Radcor 2007), October 1-5, 2007, Florence, Italy.*

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## Contents

<b>1. Introduction</b>	<b>2</b>
<b>2. Factorization of QCD amplitudes</b>	<b>2</b>
<b>3. Hadro-production of heavy quarks</b>	<b>4</b>
<b>4. Conclusions</b>	<b>7</b>

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## 1. Introduction

Amplitudes for hard scattering processes in Quantum Chromodynamics (QCD) are of basic importance both for theory and phenomenology and precision predictions for them must include higher-order quantum corrections. An important aspect in explicit computations are the singular limits of amplitudes at higher loops. Here, one has to consider two types of limits, soft and collinear, related to the emission of gluons with vanishing energy and to collinear parton radiation off massless hard partons, respectively.

For massless QCD amplitudes the corresponding singularities are regularized by working in  $d$  dimensions and appear as explicit poles in  $(d - 4)$ . Typically two powers in  $1/(d - 4)$  are generated per loop. When massive particles are involved, some of the collinear singularities are screened by the parton masses, which gives rise to large logarithmically enhanced contributions. In both cases the structure of the singularities can be understood from the factorization property of QCD and it can be predicted to all orders based on a small number of perturbatively calculable anomalous dimensions. Moreover, factorization gives rise to an extremely simple universal multiplicative relation between a massless amplitude and its massive version in the limit when the parton masses are small with respect to all other kinematical invariants. This relation can be employed to derive virtual QCD corrections at higher loops including all logarithms in the heavy quark mass as well as all constant (mass-independent) contributions. We demonstrate the predictive power of factorization with recent results for the two-loop QCD amplitudes for heavy-quark production in hadronic collisions.

## 2. Factorization of QCD amplitudes

We are interested in general  $2 \rightarrow n$  scattering processes of partons  $p$

$$p : \quad p_1 + p_2 \rightarrow p_3 + \cdots + p_{n+2}. \quad (2.1)$$