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## BCFT and Ribbon Graphs as tools for open/closed string dualities

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### Abstract

In the framework of simplicial models, we construct and we fully characterize a scalar boundary conformal field theory on a triangulated Riemann surface. The results are analysed from a string theory perspective as tools to deal with open/closed string dualities.

Simplicial techniques have been subject of renewed attention since it was clarified that they can play a role in the worldsheet description of open/closed string dualities [1]. In this connection, the hypothesis that the link between SYM gauge and closed string worldsheet dynamics could be explained through the combinatorial data associated to Strebel differentials, suggests that we are undercovering some deep discrete foundation of string dualities, which however is still far from being understood.

Aiming to give further insights in this topic, we decided to explore the connection between combinatorial data and string dualities from a more general standpoint. We considered a geometrical set-up in which ribbon graphs are dual to triangulations with localized curvature defects. These provide a natural order parameter which allows to map a  $N$ -punctured closed Riemann surface

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into an open one with  $N$  boundary components. An example of such a mapping has been given, in an hyperbolic setting, in [2]. A different construction was obtained describing a random Regge triangulation (RRT) [3] as the uniformization of an open Riemann surface  $M_\partial$  with a set of  $N$  annuli,  $\Delta_{\varepsilon(p)}^*$ ,  $p = 1, \dots, N$  each of which is defined in the neighborhood of the  $p$ -th vertex of the original triangulation. Each annulus is endowed with a correspondent Euclidean cylindrical metric and, via a conformal mapping can be equivalently interpreted as a cylinder of *finite height*. The decorated Riemann surface is subsequently constructed glueing the above local uniformizations along the pattern defined by the ribbon graph baricentrically dual to the parent triangulation.

This geometrical setup, which trades the localised curvature degrees of freedom of the parent triangulation into modular data of the new discrete surface, is simpler than that analysed in [2], but it can be dynamically coupled with matter field theory. In [4] we showed that this leads to the definition of a new kinematical background in which it could be possible to investigate dynamical processes typical of open/closed string dualities.

## 1 Boundary Insertion Operators

The geometry we dealt with is characterized by  $N$  cylinders of finite heights, which can be interpreted as open string worldsheet, connected through their inner boundary to a ribbon graph. Hence, the latter is the natural *locus* where  $N$  copies of a given Boundary Conformal Field Theory, each defined on a single cylindrical end, interact.

The quantization of a BCFT on a cylindrical domain is a delicate issue relying on the knowledge of the data associated to the correspondent *bulk* CFT. This is uniquely characterized by two copies of a chiral algebra,  $\mathcal{W}$  and  $\overline{\mathcal{W}}$ . Their generators, respectively  $W_n^a$  and  $\overline{W}_n^a$ , are the Laurent's modes of the holomorphic and antiholomorphic chiral fields of the theory, which we will call  $W^a(\zeta)$  and  $\overline{W}^a(\bar{\zeta})$  respectively. The extension of such a bulk CFT to a BCFT on  $\Delta_{\varepsilon(p)}^*$  consists in the choice of a boundary condition  $A(p)$  on  $\partial\Delta_{\varepsilon(p)}^*$ . This is usually done by specifying a glueing automorphism,  $\Omega_{A(p)}$ , which, relating the holomorphic and anti-holomorphic chiral fields on the boundary itself, avoids flux of informations through it. This process allows to define, out of  $W^a(\zeta)$  and  $\overline{W}^a(\bar{\zeta})$ , a single chiral field,  $\mathbb{W}_{\Omega(p)}(\zeta)$ . This is continuous on the full complex plane and it contains the information about the boundary condition. Its Laurent modes close a single copy of the chiral algebra, whose irreducible representations define the Hilbert space of states of the boundary theory.

In the framework dual to a RRT, each  $(p, q)$ -edge of the ribbon graph is connected to two cylindrical ends,  $\Delta_{\varepsilon(p)}^*$  and  $\Delta_{\varepsilon(q)}^*$ . Hence, its oriented boundaries are decorated by two different boundary conditions, say  $A(p)$  and  $B(q)$  respectively. In this picture, we do not have a jump between two boundary conditions taking place in a precise boundary's point. Therefore, we cannot apply standard BCFT rules which, assuming the existence of a vacuum state not invariant under translations, allow a boundary condition to change along a single boundary component.

In order to mediate between *adjacent* boundary conditions, we have thus introduced a different process which, restoring the flux of information through the boundaries connected to the ribbon graph, allows to describe the interaction