

D-term Uplifted Racetrack Inflation

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Abstract.

It is shown that racetrack inflation can be implemented in a moduli stabilisation scenario with a supersymmetric uplifting D -term. The resulting model is completely described by an effective supergravity theory, in contrast to the original racetrack models. We study the inflationary dynamics and show that the gaugino condensates vary during inflation. The resulting spectral index is $n_s \approx 0.95$ as in the original racetrack inflation model. Hence extra fields do not appear to alter the predictions of the model. An equivalent, simplified model with just a single field is presented.

Keywords: string theory and cosmology, inflation

1. Introduction

With the advent of precision cosmological data, the need for a precise description of inflation is pressing. This is all the more conspicuous in the light of the forthcoming launch of the Planck satellite. Unfortunately, inflation model building has produced a plethora of viable scenarios. Few of them, though, have a sound physical motivation. Yet it is clear that inflation belongs to the realm of high energy physics, as it takes place at energies beyond the TeV scale. Moreover, a period of inflation cannot be obtained with just the standard model fields and their interactions. It may thus be argued that a tenable determination of the inflation parameters, leading to predictions for the CMB spectrum, must be achieved within our models of physics beyond the standard model. A most promising candidate for this is string theory.

Models of inflation within string theory have been constructed recently. They all face the closely related moduli- [1] and η -problem [2]. All string moduli, with the exception of the inflaton fields, need to be fixed during inflation. The resulting inflation potential should also be sufficiently flat to obtain a period of slow-roll inflation. String models of slow roll inflation fall into two categories. The first one, brane inflation, uses the motion of branes to achieve inflation [3]. Warped throats [1] and non-relativistic motion [4, 5] may be invoked to make the inflaton potential suitably flat. The second category is moduli inflation [6, 7, 8, 9]. In this set-up one of the many string moduli fields plays the role of the inflaton. Near an extremum, the potential may be sufficiently flat for inflation. The model proposed in this paper is an example of the latter approach.

The first explicit construction of a string vacuum with all the moduli stabilised and with a positive cosmological constant is the KKLT set-up [10]. In this approach a flux potential stabilises the dilaton and the complex structure moduli. The Kähler moduli acquire a non-trivial potential from non-perturbative effects such as gaugino condensation. In general, the resulting configuration is a stable AdS vacuum with a negative cosmological constant and no supersymmetry-breaking. The lifting to a Minkowski or dS background, in which SUSY is broken, is achieved with antibranes located at the tip of the warped throats induced by the internal fluxes. Unfortunately, these antibranes break supersymmetry explicitly.

Within the KKLT framework, and assuming that the non-perturbative potential is of the racetrack type, it has been shown that saddle points in the moduli potential exist around which slow roll inflation can occur [8, 9]. One of the successes of these racetrack inflation models is the value of the spectral index $n_s \approx 0.95$, very close to the WMAP3 results. A drawback of these models is the explicit breaking of supersymmetry, which means that one must go beyond the supergravity approximation in which radiative corrections are under control. In this paper we avoid the problem by building a racetrack model where the lifting potential is due to a manifestly supersymmetric D -term. The use of uplifting D -terms has been advocated in [11, 12].

Special care must be paid to gauge invariance, anomaly cancellation and the presence of extra meson fields when constructing the gaugino condensates. It will be interesting to see how these meson fields can affect the CMB predictions of the model. This touches upon another motivation for our work, which is to test the robustness of the prediction for the spectral index $n_s \approx 0.95$. Is this a generic prediction of inflation with racetrack superpotentials, or rather a more model dependent prediction? As we will see, our model also gives $n_s \approx 0.95$. Although this is not a definite answer to the above question, it does suggest that the prediction is generic. This claim is supported by the analysis of a three cosine inflation model which is deduced from the racetrack model by freezing all the fields at their saddle point values and leaving the imaginary part of the Kähler modulus to roll slowly along the unstable direction. In particular, the spectral index of the simplified model as a function of the only free parameter η_{sad} , the η parameter at the saddle point, gives a very accurate description of the exact result obtained using the full scalar potential of the racetrack model. In this sense, the

universality of the $n_s \leq 0.95$ bound follows from the corresponding bound obtained for the three cosine model.

The paper is organised as follows. In section 2 we discuss the ingredients of the moduli sector, and present our inflation model. After a short recapitulation of the slow roll formalism in section 3, we give the results of our numerical analysis in section 4. We find that they are very similar to those of other racetrack models. This leads us to propose the simplified effective model of section 5. Our results are summarised in section 6.

2. The racetrack model

In this section we focus on the model building aspects of the D -term uplifted racetrack model of inflation. Numerical results and comparisons with the original racetrack inflation model can be found in the following sections. The idea behind the racetrack model of inflation is that the moduli sector of type IIB string theory compactified down to 4D plays the role of the inflation sector. In the original racetrack inflation papers, the procedure was implemented within the KKLT scheme whereby the vacuum of the theory is obtained via a non-supersymmetric uplifting procedure [8, 9]. In particular, uplifting is achieved using anti-D-branes living at the tip of a warped throat. One of the drawbacks of these racetrack models is the absence of an explicit supergravity description for the uplifting mechanism. In this section we will describe a supersymmetric uplifting model extending [12].

Let us first describe a minimal supergravity setting wherein the number of fields has been reduced to only three fields T and Φ_i with $i = 1, 2$. Typically, in type IIB string models, (the real part of) the modulus T describes the size of the compactification manifold while the Φ_i correspond to meson fields on D7 branes. The dynamics are described by a non-perturbative superpotential. In the scenario of [12], this potential arises from gaugino condensation on D7 branes. Racetrack inflation requires a superpotential with at least two exponential terms, which necessitates the presence of two non-Abelian gauge groups $SU(N_i)$. Considering the case of meson fields Φ_i arising from N_{fi} quark flavours of each group

$$W = W_0 + A \frac{e^{-aT}}{\Phi_1^{r_1 a}} + B \frac{e^{-bT}}{\Phi_2^{r_2 b}}. \quad (1)$$

Without loss of generality, we can take the parameters W_0, A, B to be real. The constant term arises from integrating out the stabilised dilaton and the complex structure moduli. For simplicity we are considering diagonal quark condensates $\Phi_i^2 = (1/N_{fi}) \sum_{a=1}^{N_f} Q_{ai} \bar{Q}_{ai}$, where Q_{ai} and \bar{Q}_{ai} are quark and antiquark fields.

The constants $a, b, r > 0$ are given by

$$a = \frac{4\pi k_1}{N_1 - N_f}, \quad b = \frac{4\pi k_2}{N_2 - N_f}, \quad r_i = \frac{N_{fi}}{2\pi k_i}, \quad (2)$$

where $k_i = \text{O}(1)$ are parameters relating the gauge coupling function of the $SU(N_i)$ group to the modulus field T . We have identified

$$f_i = \frac{k_i T}{2\pi} \quad (3)$$

for any gauge group G_i . The constants in front of the exponents are related to the gauge parameters by

$$A = (N_1 - N_{f1})(2N_{f1})^{r1a/2}, \quad B = (N_2 - N_{f2})(2N_{f2})^{r2b/2}. \quad (4)$$

The above effective supergravity level description is valid for $\text{Re } T \gg 1$ corresponding to the weak coupling limit.

The model possesses a pseudo-anomalous gauged $U(1)_x$ symmetry, whose anomaly is cancelled by the Green-Schwarz mechanism. This makes use of the explicit form of the $U(1)_x$ gauge coupling function $f_x = k_x T / 2\pi$, which is typical for gauge groups located on D7 branes. Gauge invariance implies that the fields transform as

$$\delta T = \eta^T \epsilon = i \frac{\delta_{\text{GS}}}{2} \epsilon, \quad \delta \Phi_i = \eta^i \epsilon = -i \frac{\delta_{\text{GS}}}{2r_i} \Phi_i \epsilon, \quad (5)$$

with ϵ the infinitesimal gauge parameter, and δ_{GS} the Green-Schwarz parameter. With the minimal field content consisting of quarks and antiquarks with $U(1)_x$ charges q_i and \bar{q}_i , the $U(1)_x SU(N_i)^2$ anomaly conditions read

$$\delta_{\text{GS}} = \frac{N_{f1}}{2\pi k_1} (q_1 + \bar{q}_1) = \frac{N_{f2}}{2\pi k_2} (q_2 + \bar{q}_2). \quad (6)$$

Using the charge assignment of the meson fields $q_{\Phi_i} = (q_i + \bar{q}_i)/2$ these anomaly conditions are automatically satisfied for the parameters (2). In addition there is the non-trivial $U(1)_x^3$ condition

$$\delta_{\text{GS}} = \frac{1}{3\pi k_x} [N_1 N_{f1} (q_1^3 + \bar{q}_1^3) + N_2 N_{f2} (q_2^3 + \bar{q}_2^3)] \quad (7)$$

which can be satisfied for suitable parameter choices.

So far, we have only alluded to the necessary stringy ingredients which lead to our low energy supergravity model. Although we do not attempt to provide a complete stringy construction of the model, more details about the possible embedding in string theory can be provided. In particular, explicit models may require the existence of more fields than the ones presented so far in a minimal setting.

First of all, the existence of chiral fields can be obtained by considering magnetised D7 branes in orientifold models [13]. The internal magnetic field is related to the effective Fayet-Iliopoulos term E/X^3 [see (10) and (14)] of the $U(1)_x$ gauge symmetry. Close to a fixed point and considering 2 stacks of N_1 and N_2 D7 branes together with a single isolated brane, we can construct the low energy spectrum which comprises the quarks Q_i in the N_i representation corresponding to the open string between the N_i D7 branes and the $U(1)_x$ brane. Antiquarks are associated to open strings between the stacks of D7 branes and the orientifold image of the $U(1)_x$ brane (hence $N_{fi} = 1$ here). There is also a field ζ associated with the open string between the $U(1)_x$ brane and its orientifold image. We focus on models where its $U(1)_x$ charge is positive implying that ζ picks up

a positive mass thanks to the effective Fayet-Iliopoulos term, and is stabilised at the origin of field space. There are also charged fields corresponding to the open string between the D7 stacks (and also between the stacks and their orientifold images).

We focus on $SU(N_i)$ D -flat directions. Flat directions are in one to one correspondence with analytic gauge invariants [14] such as the meson fields Φ_i built from the quarks and antiquarks. We consider the particular direction whereby all the analytic gauge invariants vanish except the mesons Φ_i . We specialise even further to the D -flat directions parameterised by $\Phi = \Phi_1 = \Phi_2$ (gauge invariance then requires we set $r_1 = r_2 = r$). Along this particular direction, non-perturbative phenomena in the $SU(N_i)$ gauge groups lead to the appearance of the gaugino condensation superpotential we have used. We will not pursue any further the string construction of the model. In particular, the analysis of the anomaly cancellation is modified by the presence of ζ and of the open strings linking the D7 stacks. This is left for future work.

The full potential of the theory is given by $V = V_F + V_D$ with

$$V_F = e^K \left(K^{IJ} D_I W D_J \bar{W} - 3|W|^2 \right), \quad V_D = \frac{1}{2 \operatorname{Re}(f_x)} (i\eta^I K_I)^2, \quad (8)$$

where I, J are summed over T, Φ . We will be using the superpotential (1), but with just a single meson field for simplicity (so $\Phi_i = \Phi$ and $r_i = r$). The last ingredient of the supergravity model needed to calculate it is the Kähler potential K . For gaugino condensation on D7 branes, we have a minimal Kähler for the meson fields, as in [12]

$$K = -3 \log(T + \bar{T}) + |\Phi|^2. \quad (9)$$

Defining $T = X + iY$ and $\Phi = |\phi| \dagger$ the D -term is then

$$V_D = \frac{E}{X^3} \left(1 + \frac{2X\phi^2}{3r} \right)^2 \quad (10)$$

where $E = 9\delta_{\text{GS}}^2 \pi / (16k_x)$. The F -terms give

$$\begin{aligned} V_F = & \frac{e^{\phi^2}}{24X^3} \left\{ A^2 \frac{e^{-2aX}}{\phi^{2(1+ar)}} \left(3a^2r^2 + 2a[2aX^2 + 6X - 3r]\phi^2 + 3\phi^4 \right) \right. \\ & + B^2 \frac{e^{-2bX}}{\phi^{2(1+br)}} \left(3b^2r^2 + 2b[2bX^2 + 6X - 3r]\phi^2 + 3\phi^4 + 3W_0^2\phi^2 \right) \\ & + 2AB \frac{e^{-(a+b)X}}{\phi^{2+(a+b)r}} \left(3abr^2 + [4abX^2 + 6(a+b)X - 3(a+b)r]\phi^2 + 3\phi^4 \right) \cos([a-b]Y) \\ & + 6W_0A \frac{e^{-aX}}{\phi^{ar}} (2aX - ar + \phi^2) \cos(aY) \\ & \left. + 6W_0B \frac{e^{-bX}}{\phi^{br}} (2bX - br + \phi^2) \cos(bY) \right\}. \end{aligned} \quad (11)$$

The kinetic terms obtained from the Kähler potential are

$$\mathcal{L}_{\text{kin}} = \frac{3}{4X^2} (\partial_\mu Y \partial^\mu Y + \partial_\mu X \partial^\mu X) + \partial_\mu \phi \partial^\mu \phi. \quad (12)$$

\ddagger The Goldstone boson $\arg \Phi$ can be gauged away, and becomes the longitudinal polarisation of the anomalous $U(1)_x$ vector field. At the level of the scalar potential it corresponds to a flat direction in V .