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Boundary Correlators in Supergroup WZNW Models

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Abstract

We investigate correlation functions for maximally symmetric boundary conditions in the WZNW model on GL(1|1). Special attention is payed to volume filling branes. Generalizing earlier ideas for the bulk sector, we set up a Kac-Wakimoto-like formalism for the boundary model. This first order formalism is then used to calculate bulk-boundary 2-point functions and the boundary 3-point functions of the model. The note ends with a few comments on correlation functions of atypical fields, point-like branes and generalizations to other supergroups.

Contents

1 Introduction

Sigma models on supergroups and their cosets are an interesting subject of current research. They occur in a number of very different problems ranging from string theory to disordered electron systems. In addition to such concrete applications, conformal field theories with target space supersymmetry may also be studied for their structural and mathematical properties. They provide examples of non-unitary models, many of which have vanishing or negative central charge. Moreover, their correlation functions often possess logarithmic singularities. As shown in [1], both properties are intimately related to features of the supergroup geometry.

The simplest non-trivial model to consider is the WZNW model on the supergroup GL(1|1). Studies of this field theory go back to the work of Rozanski and Saleur [2, 3]. These early investigations of the GL(1|1) WZNW model stimulated much further work on the emerging topic of logarithmic conformal field theory (see e.g. [4, 5] for a review). A few years back, the GL(1|1) WZNW model was revisited in [1] from a geometric rather than algebraic perspective. Based on the harmonic analysis of the supergroup GL(1|1), a proposal was formulated for the exact spectrum of the field theory. Furthermore, efficient computational tools were developed to calculate correlation functions of tachyon vertex operators. Finally, the consistency of the proposed spectrum was demonstrated explicitly.

The work [1] was restricted to the GL(1|1) WZNW model on the sphere, i.e. neither boundaries nor higher genus surfaces were included. Subsequent work [6] extended part of the bulk analysis to the boundary sector. In particular, the geometric interpretation of maximally symmetric boundary conditions was unravelled. This led to several proposals for the spectra of boundary operators in the corresponding boundary conformal field theories. These were tested partially through the so-called modular bootstrap. Correlation functions with non-trivial insertions of bulk and boundary operators were not computed in [6]. We are now aiming to close this gap, at least for one type of boundary conditions.

There are several motivations to determine boundary correlation functions in supergroup WZNW models. To begin with, the conjectured boundary spectra in [6] contained information that cannot be probed through the modular bootstrap alone. In particular, certain boundary correlation functions were predicted to contain logarithmic singularities. Below we shall be able to verify such features of the boundary conformal field theory. Moreover, 2-dimensional boundary field theories are intimately related with quantization theory (see e.g. [7, 8, 9, 10] and references therein). While the GL(1|1) WZNW model itself is a bit too simple to accommodate for interesting supersymmetric extensions of non-commutative geometry, the methods we shall develop below possess generalizations to cases with a curved bosonic base. The latter provide a much richer geometric framework, with further links to representation theory of affine algebras and the quantization of Lie superalgebras. Finally, let us also mention possible applications to the study of branes and open strings in superspaces, and in particular to AdS backgrounds.

To be a bit more specific about the results we are going to obtain, we recall from [6] that there are two different families of maximally symmetric boundary conditions in the GL(1|1) WZNW model. Geometrically, the first set consists of D-branes that are point-like localized in the bosonic base. They extend into both fermionic directions, unless they are placed along very special lines in the base manifold. The second set of boundary conditions contains a single object: a volume filling brane that extends in all bosonic and fermionic directions. We called this brane *twisted* because it is associated with the only non-trivial gluing automorphism of the current algebra. In [6], some simple amplitudes for the point-like D-branes have been computed. On the other hand, the methods of [6] were not sufficient to obtain non-trivial amplitudes for the volume filling brane.

In this work we shall extend some of the techniques from [1] to compute correlation functions of bulk and boundary operators for the volume filling brane. The main results include explicit formulas (4.2,4.7,4.9) for the bulk-boundary 2-point function and (4.16-4.19) for the boundary 3-point functions. The information they contain is equivalent to the bulk-boundary and the boundary operator product expansion, respectively. Our results provide a complete solution of the boundary theory for the volume filling brane. We shall also determine a non-trivial annulus amplitude.

In order to obtain these results we set up a first order formalism for the volume filling brane. It is obtained by adding an appropriate square root of the bulk interaction term along the boundary of the world-sheet. As in other theories containing fermions, taking the square root forces us to introduce an auxiliary fermion along the boundary. All this will be explained in great detail in section 2. A perturbative expansion for correlators of the boundary conformal field theory is set up in section 3. It is employed in Section 4 to solve explicitly the boundary GL(1|1) WZNW model with twisted boundary conditions. Section 5 contains an alternative approach to computing amplitudes that involve only special (atypical) fields/states of the theory. It is used to prove that the GL(1|1) WZNW contains a special subsector whose correlation functions are independent of the level k. The second approach is finally employed to compute a particular annulus amplitude for the volume filling brane. The latter provides a nice test for the boundary state that was proposed in [6]. We conclude with a list of open problems, mostly related to the point-like branes for GL(1|1) and extensions to higher supergroups.

2 Volume filling brane: The classical action

Our aim in this section is to discuss the classical description of volume filling branes in the GL(1|1) WZNW model. To begin with, we spell out the standard action of the

WZNW model with so-called twisted boundary conditions. Their geometric interpretation as volume filling branes with a non-zero B-field is recalled briefly. In order to set up a successful computation scheme for the quantum theory later on, we shall need a different formulation of the theory. As in the bulk theory, computations of correlations functions require a Kac-Wakimoto like representation of the model [1]. Finding such a first order formalism for the boundary theory is not entirely straightforward. We shall see that it requires introducing an additional fermionic boundary field.

2.1 The boundary WZNW model

Following our earlier work on WZNW models for type I supergroups, we parametrize the supergroup GL(1|1) through a Gauss-like decomposition of the form

$$g \;=\; e^{i\eta_- \psi^-}\, e^{ixE+iyN}\, e^{i\eta_+\psi^+}$$

where E, N and $\psi^\pm$ denote bosonic and fermionic generators of gl(1|1), respectively. In the WZNW model, the two even coordinates x, y become bosonic fields X, Y and similarly, two fermionic fields $c_\pm$ come with the odd coordinates $\eta_\pm$. Let us now consider a boundary WZNW model with the action

$$\begin{aligned} S_{\text{WZNW}}(X,Y,c_\pm) \;&=\; -\frac{k}{4\pi i}\int_\Sigma d^2z\; \left(\partial X\bar\partial Y + \partial Y\bar\partial X + 2e^{iY}\partial c_+\bar\partial c_-\right) + \\ &\quad + \frac{k}{8\pi i}\int du\; e^{iY}(c_+ + c_-)\partial_u(c_+ + c_-)\;, \end{aligned} \tag{2.1}$$

where u parametrizes the boundary of the upper half plane. Variation of the action leads to the usual bulk equations of motion along with the following set of boundary conditions

$$\begin{aligned} \partial_v Y \;=\; 0 \quad , \quad 2\partial_v X \;&=\; e^{iY}(c_+ + c_-)\,\partial_u(c_+ + c_-)\;, \\ \pm 2\partial_v c_\pm \;&=\; 2i\partial_u c_\mp - (c_- + c_+)\,\partial_u Y\;. \end{aligned} \tag{2.2}$$

Here, we have used the derivatives $\partial_u = \partial + \bar\partial$ and $\partial_v = i(\partial - \bar\partial)$ along and perpendicular to the boundary. The equations (2.2) imply Neumann boundary conditions for all four fields of our theory, i.e. we are dealing with a volume filling brane. Since the normal derivatives of the fields X and $c_\pm$ do not vanish, our brane comes equipped with a B-field. A more detailed discussion of the brane's geometry can be found in our recent paper [6].