Konrad-Zuse-Zentrum
für Informationstechnik Berlin

Takustraße 7
D-14195 Berlin-Dahlem
Germany

ANDREAS BLEY
THORSTEN KOCH
LINGFENG NIU

Experiments with nonlinear extensions to SCIP

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Experiments with nonlinear extensions to SCIP*

Andreas Bley Thorsten Koch Lingfeng Niu†

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Abstract

This paper describes several experiments to explore the options for solving a class of mixed integer nonlinear programming problems that stem from a real-world mine production planning project. The only type of nonlinear constraints in these problems are bilinear equalities involving continuous variables, which enforce the ratios between elements in mixed material streams.

A branch-and-bound algorithm to handle the integer variables has been tried in another project. However, this branch-and-bound algorithm is not effective for handling the nonlinear constraints. Therefore state-of-the-art nonlinear solvers are utilized to solve the resulting nonlinear subproblems in this work. The experiments were carried out using the NEOS server for optimization. After finding that current nonlinear programming solvers seem to lack suitable preprocessing capabilities, we preprocess the instances beforehand and use a heuristic approach to solve the nonlinear subproblems.

In the appendix, we explain how to add a polynomial constraint handler that uses IPOPT as embedded nonlinear programming solver for the constraint programming framework SCIP. This is one of the crucial steps for implementing our algorithm in SCIP. We briefly describe our approach and give an idea of the work involved.

1 Introduction

Mixed Integer Nonlinear Programming (MINLP) is an optimization problem with continuous and discrete variables and nonlinearities in the objective function or at least one of the constraint functions. Many real world applications are suitable to be modeled as MINLP, because it can simultaneously optimize the system structure (discrete) and parameters (continuous). One particular type of nonlinear constraints that is often encountered are mixing or blending constraints, which enforce that the mixing ratio of materials is

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†State Key Laboratory of Scientific and Engineering Computing, Institute of Computational Mathematics and Scientific/Engineering Computing, AMSS, CAS, Beijing

the same in certain material streams. Mathematically, these mixing constraints can be expressed as bilinear equations. In this paper, we consider solving a special mixed integer nonlinear programming problem arising from the problem of scheduling the production of an open-pit mine, whose nonlinear constraints are just bilinear equations.

1.1 Problem Description

The mine can be considered as a set of *panels*, which are identified by the numbers $1, \ldots, N$. Each panel represents a volume of underground material. In particular, there are a number of mineral *attributes* which are of interest in the mining operation. We denote the set of all attributes of interest by A. For each panel, the quantity of attributes in A is assumed to be known. The value $\alpha_i^a \in \mathbb{Z}_+$ denotes the quantity of attributes $a \in A$ in panel $i \in \{1, \ldots, N\}$. A special attribute is *rock*, which describes the total tonnes of underground material in each panel.

In the operation of the mine, *rock* is *extracted* from the mine and then can be sent to one of three destinations: *waste*, *processing*, or into a *stockpile*. Material that is sent to waste can be ignored in scheduling production. Processing is performed at a processing plant that extracts the valuable material from the rock. It is assumed that any material processed is immediately sold. All material put into the stockpile is immediately mixed, thus becoming homogeneous. Nothings happens in the stockpiles, so their attributes are simply the sum of those of their constituent ingredients. At any stage, the material on the stockpile can be sent to processing.

An important feature of an open-pit mining operation is the *precedence* structure. For each panel $i = 1, \ldots, N$, mining engineering software is able to calculate the set of other panels, denoted by $\text{Pred}(i) \subseteq \{1, \ldots, N\}$, that must be completely extracted before the extraction of panel i can safely begin.

The profitability of a mine is calculated as its net present value (NPV), which depends crucially on *when* the valuable material extracted is sold. Thus the life of the mine is divided into T periods. The mine *schedule* specifies the mining activities, i.e., the material to be extracted, processed and stockpiled in each period. The profit that these activities yield must be multiplied by a period discount factor, which we denote by δ_t for each $t = 1, \ldots, T$, and the sum of the profits, weighted by the discount factor for the period in which they occur, gives the NPV. We seek a schedule that maximizes this NPV.

In this specific application, the nonlinearities arise from the use of stockpiles. In fact, the open pit mine production planning problem without the use of stockpiles can be formulated and solved very efficiently as mixed integer linear programming problems. Several variants of such problems are described in [13, 12, 6]. We will introduce a MINLP formulation for a mine with a single infinite-capacity stockpile briefly in the next subsection.

1.2 MINLP formulation

In this work, we only consider three attributes: the total *rock* tonnage, the *ore* content, and the *metal* content, i.e., $A = \{rock, ore, met\}$. We make the simplifying assumption that the stockpile is empty at the start of the first time period and at the end of the planning horizon. Furthermore, we assume that material taken from the stockpile is taken off instantaneously at the start of the respective period. Material added to the stockpile is added at the end of a period, i.e., after the removal of the material taken off for processing in the same period, but before the removal of the material taken off for processing in the following period.

The following parameters and variables are used for our model description.

Parameters (all non-negative):

p_t^{met}	metal price per unit in period t
c_t^m	extraction cost per ton of rock in period t
c_t^p	processing cost per ton of ore in period t
b_t^m	extraction capacity in tons of ore in period t
b_t^p	processing capacity in tons of rock in period t

Note that the processing capacity and cost depend only on the ore tonnage of the material sent for processing, while the mining capacity and cost depend on the rock tonnage of the material extracted.

Variables

For each panel $i = 1, .., N$ and each period $t = 1, ..., T$:

$f_{i,t}^m \in [0, 1]$	fraction of panel i extraced in period t
$f_{i,t}^p \in [0, 1]$	fraction of panel i sent directly for processing in period t
$f_{i,t}^s \in [0, 1]$	fraction of panel i sent to stockpile in period t
$f_{i,t}^o \in [0, 1]$	fraction of panel i sent from stockpile to stockpile in period t
$f_{i,t}^r \in [0, 1]$	fraction of panel i remaining in stockpile from period $t-1$ to period t

In order to ensure that the extraction respects the given precedence relations required for safe mining, we introduce binary decision variables:

$$x_{i,t} = \begin{cases} 1 & \text{panel } i \text{ is completely extracted by end of period } t \text{ or earlier,} \\ 0 & \text{otherwise.} \end{cases}$$

Then the **M**ine **P**roduction **S**cheduling problem with a single **S**tockpile (MPSS) can be formulated as follows:

$$\max \textstyle\sum_{t=1}^{T} \delta_t (\sum_{i=1}^{N} ((p_t^{met} \alpha_i^{met} - c_t^p \alpha_i^{ore})(f_{i,t}^p + f_{i,t}^o) - c_t^m \alpha_i^{rock} f_{i,t}^m)) \qquad (1.1)$$

$$
\begin{aligned}
\text{s.t.} \qquad & f^p_{i,t} + f^s_{i,t} \le f^m_{i,t}, && \forall i = 1, ..., N, t = 1, ..., T. \\
& \textstyle\sum_{t=1}^T f^m_{i,t} \le 1, && \forall i = 1, \cdots, N. \\
& \textstyle\sum_{t'=1}^t f^m_{i,t'} \ge x_{i,t}, && \forall i = 1, ..., N, t = 1, ..., T. \\
& \textstyle\sum_{t'=1}^t f^m_{i,t'} \le x_{j,t}, && \forall i = 1, ..., N, t = 1, ..., T, j \in \text{Pred}(i). \\
& x_{i,t} \le x_{i,t+1}, && \forall i = 1, ..., N, t = 1, ..., T-1. \\
& f^r_{i,t} + f^s_{i,t} = f^o_{i,t+1} + f^r_{i,t+1}, && \forall i = 1, \cdots, N, t = 1, \cdots, T-1. \\
& \textstyle\sum_{i=1}^N \alpha_i^{rock} f^m_{i,t} \le b^m_t, && \forall t = 1, ..., T. \\
& \textstyle\sum_{i=1}^N \alpha_i^{ore} (f^p_{i,t} + f^o_{i,t}) \le b^p_t, && \forall t = 1, ..., T. \\
& f^o_{i,t} f^r_{j,t} = f^o_{j,t} f^r_{i,t}, && \forall i, j = 1, \cdots, N, t = 1, \cdots, T. \\
& 0 \le f^m_{i,t}, f^s_{i,t}, f^p_{i,t}, f^o_{i,t}, f^r_{i,t} \le 1, && \forall i = 1, ..., N, t = 1, ..., T. \\
& f^o_{i,1} = f^r_{i,1} = f^r_{i,T} = f^s_{i,T} = 0, && \forall i = 1, \cdots, N. \\
& x_{i,t} \in \{0, 1\}, && \forall i = 1, ..., N, t = 1, ..., T,
\end{aligned}
$$

where the bilinear constraint $f^o_{i,t} f^r_{j,t} = f^o_{j,t} f^r_{i,t}$ represents the requirement that all the material in the stockpile must be homogeneously mixed. This MINLP contains $5NT$ continuous variables and NT binary variables. The number of nonlinear constraints is NT. For a mine with 125 panels and 25 time periods (this is the scale of one of our experiment problems), the scale of the model is 3125 binary variables and 390625 bilinear constraints. Since this is considered a large scale instance regarding current MINLP solving techniques, we would prefer a more compact formulation.

Notice that variables $f^o_{\cdot,\cdot}$ and $f^r_{\cdot,\cdot}$ are only used to indicate the material flows in and out of the stockpile. Now consider using the following aggregated variables instead of reducing the number of variables and nonlinear constraints . We define for each period $t = 2, ..., T$ and each attribute $a \in \{ore, met\}$ the following variables:

q^a_t units of attribute a in the stockpile at the end of period t,
o^a_t units of attribute a removed from the stockpile at the start of period t (i.e., at the end of period $t-1$).

As all material put into the stockpile is mixed, we have to ensure that the material taken off the stockpile at the start of a period has the same metal–ore composition as the material contained in the stockpile at the end of the preceding period. Therefore, the following bilinear equalities are needed,

$$o^{ore}_t q^{met}_{t-1} = o^{met}_t q^{ore}_{t-1} \qquad \forall t = 2, \ldots, T.$$

which is also the only group of nonlinear constraints that are necessary to model the mixing property of the stockpile. Then we get another MINLP model:

$$\max \sum_{t=1}^T \delta_t \Big(\sum_{i=1}^N ((p^{met}_t \alpha^{met}_i - c^p_t \alpha^{ore}_i) f^p_{i,t} - c^m_t \alpha^{rock}_i f^m_{i,t}) + p^{met}_t o^{met}_t - c^p_t o^{ore}_t \Big) \tag{1.2}$$