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The Line Connectivity Problem

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Abstract

This paper introduces the *line connectivity problem*, a generalization of the Steiner tree problem and a special case of the line planning problem. We study its complexity and give an IP formulation in terms of an exponential number of constraints associated with "line cut constraints". These inequalities can be separated in polynomial time. We also generalize the Steiner partition inequalities.

1 Introduction

The *line connectivity problem* (LCP) can be described as follows. We are given an undirected graph $G = (V, E)$, a set of *terminal nodes* $T \subseteq V$, and a set of *lines* L (simple paths) defined on the graph G, see the left of Figure 1 for an example. The lines have nonnegative costs $C \in \mathbb{R}_+^L$ and cover all edges, i.e., for every $e \in E$ there is an $\ell \in L$ such that $e \in \ell$. The problem is to find a set of lines $L' \subseteq L$ of minimal cost such that for each pair of distinct terminal nodes $t_1, t_2 \in T$ there exists a path from t_1 to t_2, which is completely covered by lines of L'.

LCP is a generalization of the Steiner tree problem (STP) since we get an STP if all lines have length one. In contrast to the STP with nonnegative costs, see [4, 5] for an overview, the optimal solution of the line connectivity problem does not have to be a tree. There can be two lines that form a cycle, but both are necessary to connect two terminal nodes, see the right of Figure 1. However, an optimal solution of LCP is minimally connected, i.e., if we remove a line from the solution, there exist at least two terminals which are not connected.

LCP is a special case of the *line planning problem* in which passenger routes are not fixed a priori, see [2] and the references therein for a detailed definition. Line planning deals with finding a set of lines and corresponding frequencies such that a given demand can be transported. Usually, the objective is to minimize cost and/or travel times. If we neglect travel time,

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