Konrad-Zuse-Zentrum
für Informationstechnik Berlin

Takustraße 7
D-14195 Berlin-Dahlem
Germany

MICHAEL ZILSKE, HANS LAMECKER[1], STEFAN ZACHOW

Adaptive Remeshing of Non-Manifold Surfaces

[1]Supported by the DFG Research Center MATHEON "Mathematics for key technologies"

accepted for publication in Proc. EUROGRAPHICS 2008 / G. Drettakis and R. Scopigno, Volume 27 (2008), Number 3

ZIB-Report 07-01 (Januar 2007)

Adaptive Remeshing of Non-Manifold Surfaces

Michael Zilske, Hans Lamecker, Stefan Zachow

January 15, 2008

Abstract: We present a unified approach for consistent remeshing of arbitrary non-manifold triangle meshes with additional user-defined feature lines, which together form a feature skeleton. Our method is based on local operations only and produces meshes of high regularity and triangle quality while preserving the geometry as well as topology of the feature skeleton and the input mesh.

ACM Computing Classification System (1998): I.3.3 [Computer Graphics]: Line and Curve Generation, I.4.6 [Computer Graphics]: Feature Detection, I.4.7 [Computer Graphics]: Feature Measurement

1 Introduction

The majority of remeshing schemes for triangular surfaces are designed for manifold-with-boundary surface models [AUGA05]. Some methods which rely on a global parameterization of the surface even require it to be isomorphic to a disk. However, there are applications where this restriction is too prohibitive. Important examples are bio-medical applications, that often require geometric models which volumetrically describe different tissue compartments, and computer-aided engineering, where interfaces between more than two adjacent machine parts are often modeled. In these cases boundary surfaces separating different materials are not represented as separate objects, but as a single mesh comprised of several surface patches. They contain *seams*, i.e. paths of non-manifold edges, where more than two different regions meet. The problem of remeshing such non-manifold surfaces, however, has - to the best of our knowledge - not been addressed.

Besides the preservation of seams another important aspect of surface remeshing is the preservation of feature lines. A feature line can be an automatically detected or interactively defined edge-path on the input surface which is expected to have a corresponding edge-path in the output mesh. Such feature lines are often required for post-processing, e.g. for surface decomposition and nurbs surface generation.

In this paper, we introduce a remeshing scheme for non-manifold surface models, which at the same time preserves feature lines embedded into the input mesh. Our method can be regarded as a generalization of [SG03, SAG03] to non-manifold surfaces with feature lines. It is based on local operations only. Our main contribution is the unified treatment of feature lines and non-manifold vertices by means of feature skeletons, which were introduced by Vorsatz, Rössl and Seidel [VRS03] specifically to preserve feature lines. It turns out that the restricted set of local operations allowed on feature lines is exactly the same set of operations which can be used to remesh seams, preserving the topology of the mesh as well as assuring geometric fidelity to the input. Furthermore, we address the problem of adapting the *vertex density*, i.e. the sampling rate, on seams and feature lines to the vertex density of the surrounding mesh, which is important if the goal is a highly regular mesh.

Local mesh operations include edge contractions and splits to modify the number of vertices in the mesh. While the topological validity of edge contraction can be easily determined in the manifold case, it is more involved for non-manifold meshes [DEGN98]. These considerations have led to a non-manifold mesh simplification scheme which additionally preserves the topology of a set of paths embedded into the input mesh as edge constraints, e.g. feature lines [VBL05].

In a remeshing scheme by Alliez et al. [AECdVDI03], an initial vertex distribution is created by means of an error diffusion methods. Precise isotropic vertex placement is achieved in a second step by constructing a weighted centroidal Voronoi diagram in a 2D parametric domain. The scheme preserves edge constraints by sampling them separately, taking special care to ensure that their sampling rate matches that of the surrounding mesh. Because this algorithm uses a global parametrization, input meshes not homeomorphic to a disk must be decomposed. The cuts are treated similar to edge constraints and are preserved in the output mesh. This is not optimal for high-genus surfaces, which require a high number of cuts.

There is more than one way to preserve edge-path-constraints in relaxation based remeshing schemes. In [VRKS01] they are ignored during vertex relaxation and later reconstructed by snapping vertices to constrained edges. This implies that equalization of the vertex density is not impeded by the constraints. It is, however, not clear how this approach could be adapted to handle non-manifold edge-paths.

The advancing front method for vertex insertion, which is common in mesh generation, has been used for remeshing [SSFS06], too. The scheme preserves edge constraints simply by using them as the initial fronts. It is suitable for interactive local remeshing and could be extended to handle non-manifold meshes.

The approach presented here is relaxation based, treating the combined set of non-manifold and constrained edges as a skeleton which is to be preserved both geometrically and topologically during remeshing.

2 Remeshing of Non-Manifold Surfaces

The input to the remeshing scheme is a triangle mesh M_0 and a set of feature lines embedded into the mesh. The algorithm proceeds by applying a series of local modifications to a mesh M, which is initialized with M_0, until it meets some quality criteria:

- Edge contractions/splits to change the number of vertices.
- Vertex movement/edge flips to improve triangle quality.

To ensure fidelity to the original mesh, it is assured that the vertices of M always remain on the original mesh M_0, and some geometric error tolerances are enforced. Their positions are determined by optimizing some quality criterion within a local neighborhood of each vertex. Good results are achieved by equalizing triangle areas [SG03] and, in later iterations, areas of the local Voronoi cells [SAG03]. Both criteria can be weighted to achieve vertex sampling corresponding to some density function, such as surface curvature. These operations are alternated with angle improving Delaunay flips. The local operations have been described for non-constrained, manifold vertices only. In the next section we explain how to adapt the set of operations to non-manifold vertices and vertices on feature lines.

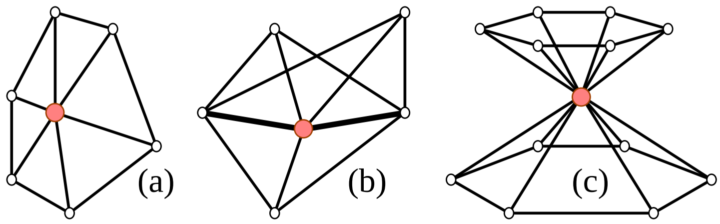

Figure 1: Vertices: (a) manifold, (b) open-book, (c) other

2.1 Feature Skeleton

A vertex is called manifold if the underlying space of its star is homeomorphic to a disk (Fig. 1a). Non-manifold vertices whose neighborhood can be arranged to form k *pages* which are joined at exactly two common edges, are called vertices with an open-book neighborhood (Fig. 1b).

Manifold vertices can be moved anywhere within their star. Vertices with an open-book neighborhood can be moved along the two common edges of its pages. Other non-manifold vertices (Fig. 1c) cannot be moved.

A vertex on a feature line can be regarded as a vertex with an open-book neighborhood consisting of exactly 2 pages. This allows to unify seams and feature lines into a single structure denoted by *feature skeleton*. The elements of the feature skeleton are further classified as follows (Fig. 2):

Skeleton edges are all non-manifold edges and all feature edges. *Skeleton vertices* are all vertices which belong to exactly two skeleton edges and additionally have the property that their star is homeomorphic to an open book. *Branching vertices* are all vertices which belong to either exactly one or more than two skeleton edges. All non-manifold vertices which are not skeleton vertices are also branching vertices.

A *skeleton segment* is a path of skeleton edges whose endpoints are branching vertices, and which does not contain any further branching vertices. Under the assumption that every closed path of skeleton edges contains at least one branching vertex, every skeleton vertex now belongs to exactly one skeleton segment. This assumption can be enforced by simply designating an arbitrary vertex as a branching vertex.

Note that every edge of a single skeleton segment belongs to the same number k of triangles. The segment is regarded as a seam of k pages (surface patches). With $k = 2$, this definition includes feature lines, and with $k = 1$ it includes the proper boundary curves of the mesh.

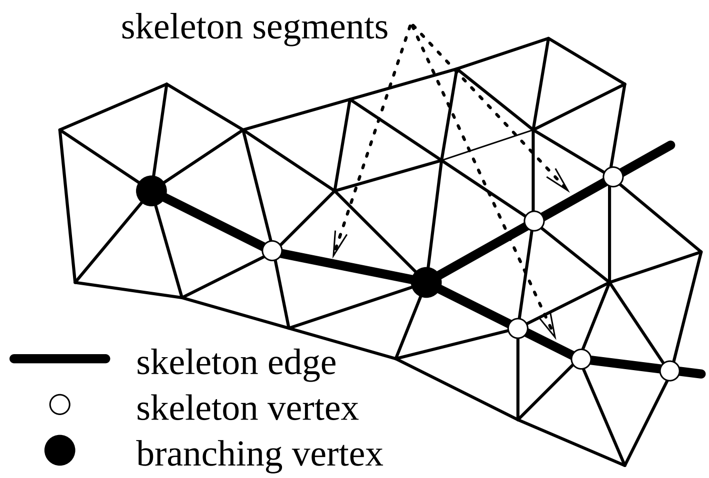

Figure 2: Feature skeleton.

2.2 Local Operations on the Feature Skeleton

Vertex Movement. We require skeleton vertices to geometrically remain on the common boundary of all k patches, so their movement is restricted to the skeleton segment. When a skeleton vertex v is moved, its new position is chosen on one of its two neighboring skeleton edges. The corresponding position on the input mesh M_0 is determined in an arc-length parameterization of the skeleton segment. Branching vertices are not moved by definition.

Edge Contraction. When contracting edges, special care must be taken to preserve the topological type of the mesh. In a manifold setting, an edge (a, b) can be contracted if and only if the common neighbors of a and b are exactly the vertices opposite (a, b) in the triangles to which (a, b) belongs. In the more general setting, this condition is necessary, but not sufficient. Edge contractions involving the feature skeleton must be restricted. We consider edge contractions where the position of the new vertex coincides with one of the original vertices, i.e. a vertex is collapsed into another vertex. A branching vertex is never collapsed into another vertex. A skeleton vertex is not collapsed into vertices outside the feature skeleton (but neighbors which are not skeleton vertices can be collapsed into the skeleton). A skeleton vertex can only be removed by contracting one of its two skeleton edges. In the special case in which a skeleton segment consists entirely of the three edges of one single triangle, none of these edges can be contracted, as this would remove a cycle from the skeleton and introduce a new branching vertex. A more formal account of topology-preserving edge-contractions is given in [DEGN98] and [VBL05].

Edge Split. When a skeleton edge e is split, the newly inserted vertex becomes a skeleton vertex, and the two new edges become skeleton edges of the same segment that e belonged to.

Edge Flip. A skeleton edge cannot be flipped (the edge flip is undefined for $k \neq 2$ anyway).

Strictly adhering to these local operations guarantees that the topology of the mesh as well as of the embedded feature skeleton is preserved.

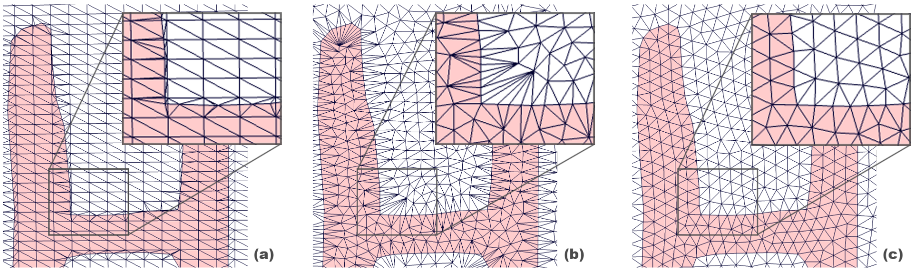

Figure 3: Improving vertex sampling along the skeleton: (a) before remeshing (b) w/o and (c) w/ connectivity regularization

2.3 Vertex Sampling on the Feature Skeleton

Applying the remeshing scheme of [SG03, SAG03] to surfaces with seams and features using the modified local operations described in section 2.2 has a significant defect.

Since vertices on the feature skeleton cannot move away from the skeleton and hence manifold vertices cannot penetrate skeleton segments, the sampling rate may differ significantly on and around the feature skeleton (see Fig. 4b).

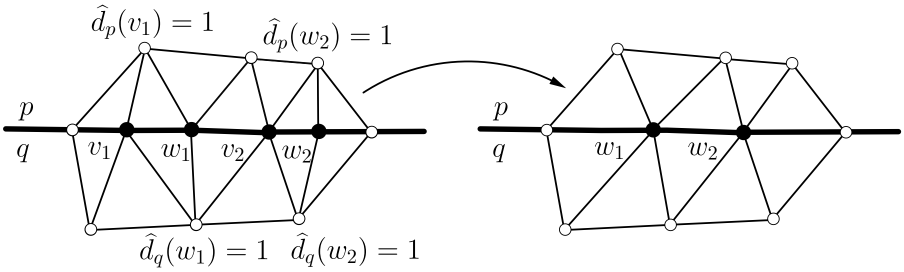

Figure 4: Contracting edges along the skeleton.

We adapt the sampling rate on the feature skeleton to its vicinity by contracting or splitting skeleton edges (Sec. 2.2) based on a connectivity criterion. On a perfectly straight feature line embedded in a perfectly regular mesh, every vertex on the line has exactly two neighbors $\widehat{d}_p$ on each side p of the line. If a sequence of vertex relaxations and edge flips cannot approximate this condition the skeleton is either sampled too densely or too sparsely. The extreme conditions where a skeleton segment is sampled far too densely produces characteristic fans of vertices with only one neighbor on each side. Contracting/splitting edges based on the following criteria turns out to be a stable way to remove these fans and adapt the sampling on the segment to the surrounding mesh: (1) Contract edge $e = (v, w)$ if $(\widehat{d}_p(v) = 1) \wedge (\widehat{d}_p(w) \leq 2) \vee (\widehat{d}_p(w) = 1) \wedge (\widehat{d}_p(v) \leq 2)$ and (2) split e if $(\widehat{d}_p(v) \geq 3) \wedge (\widehat{d}_p(w) \geq 2) \vee (\widehat{d}_p(w) \geq 3) \wedge (\widehat{d}_p(v) \geq 2)$ for each page p. After each such operation all surrounding vertices are relaxed and Delaunay edge flips are performed.

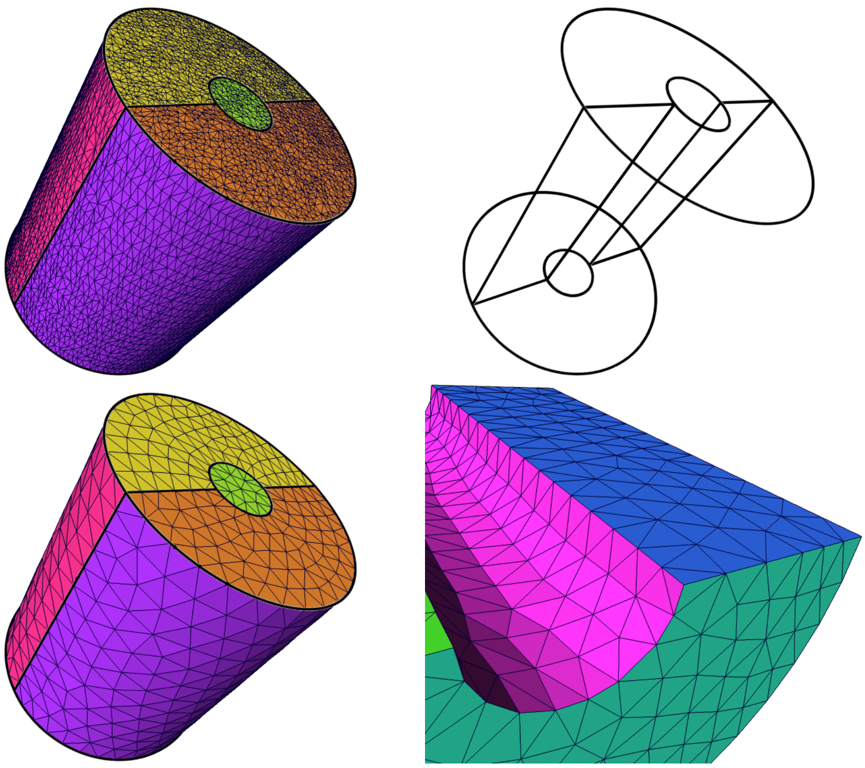

Figure 5: Remeshing of a non-manifold cylinder.

3 Results

Fig. 5 shows a uniform remeshing of a non-manifold cylinder model. The original model contains 8500, the remeshed version has 1000 vertices. The close-up shows that the sampling rate on and around the seams are balanced nicely, such that the triangles around the seams are of good quality, too.

A complex model of different anatomical regions of a bee brain is shown in Fig. 6. It contains about 50000 points and consists of many different touching materials resulting in many seams. The triangle quality as well as vertex sampling rate is very good even along the seams.

4 Conclusion

We have extended a remeshing scheme based on local operations to deal with arbitrary non-manifold meshes with additional user-specified feature lines. These are treated in a unified manner to produce meshes of high regularity and quality.

The disadvantage of using local operations for remeshing is that the vertex distribution cannot efficiently be adapted globally to a given density function, since convergence is generally very slow. Moreover, the feature skeleton inhibits global adaptation of the vertex distribution to a given density function. One idea to remedy this situation is to introduce permeable feature lines by developing a

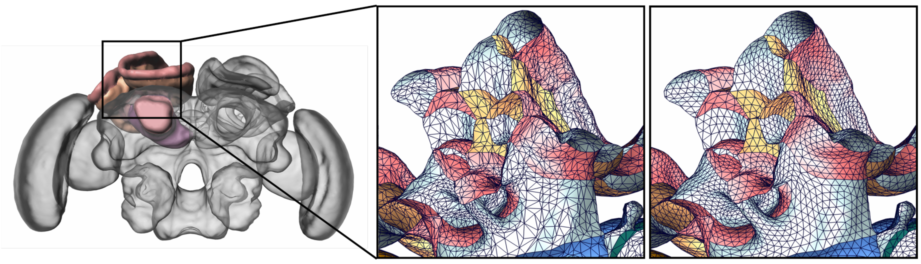

Figure 6: Remeshing of a geometric model of a bee brain with a large number of non-manifold seams.

local operation that transports a vertex from one side to the other.

References

[AECdVDI03] Alliez P., Éric Colin de Verdière, Devillers O., Isenburg M.: Isotropic surface remeshing. In *Proc. Shape Modeling International* (2003), pp. 49–58.

[AUGA05] Alliez P., Ucelli G., Gotsman C., Attene M.: Recent advances in remeshing of surfaces. *AIM@SHAPE Network of Excellence* (2005).

[DEGN98] Dey T. K., Edelsbrunner H., Guha S., Nekhayev D. V.: Topology preserving edge contraction, 1998. Technical Report RGI-Tech-98-018.

[SAG03] Surazhsky V., Alliez P., Gotsman C.: Isotropic remeshing of surfaces: a local parameterization approach. In *Proc. 12th International Meshing Roundtable* (2003).

[SG03] Surazhsky V., Gotsman C.: Explicit surface remeshing. In *Proc. Eurographics/ACM SIGGRAPH Symposium on Geometry Processing* (2003), pp. 20–30.

[SSFS06] Schreiner J., Scheidegger C., Fleishman S., Silva C.: Direct (re)meshing for efficient surface processing. *Comp. Graph. Forum 25*, 3 (2006), 527–536.

[VBL05] Vivodtzev F., Bonneau G.-P., Letexier P.: Topology preserving simplification of 2d non-manifold meshes with embedded structures. *The Visual Computer 21*, 8 (2005).

[VRKS01] Vorsatz J., Rössl C., Kobbelt L., Seidel H.-P.: Feature sensitive remeshing. In *Comp. Graph. Forum* (2001), vol. 20, pp. 393–401.

[VRS03] Vorsatz J., Rössl C., Seidel H.-P.: Dynamic remeshing and applications. In *Proc. 8th ACM symposium on Solid modeling and applications* (2003), pp. 167–175.