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Adaptive Timestep Control for the Contact–Stabilized Newmark Method[1]

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Adaptive Timestep Control for the Contact–Stabilized Newmark Method†

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Abstract

The aim of this paper is to devise an adaptive timestep control in the contact–stabilized Newmark method (ContacX) for dynamical contact problems between two viscoelastic bodies in the framework of Signorini's condition. In order to construct a comparative scheme of higher order accuracy, we extend extrapolation techniques. This approach demands a subtle theoretical investigation of an asymptotic error expansion of the contact–stabilized Newmark scheme. On the basis of theoretical insight and numerical observations, we suggest an error estimator and a timestep selection which also cover the presence of contact. Finally, we give a numerical example.

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1 Introduction

Dynamical contact problems arise in different applications such as biomechanics. In classical approaches, they are modelled via Signorini's contact conditions which are based on the non-penetration of mass. Both in analytical models and in numerical schemes, the resulting nonsmooth and nonlinear variational inequalities give rise to fundamental mathematical difficulties.

Concerning the time discretization of dynamical contact problems, the Newmark method is one of the most popular numerical integrators. As it is well-known, the classical scheme may lead to artificial numerical oscillations at dynamical contact boundaries, and even an undesirable energy blow-up during time integration may occur [6, 19]. In [13], Kane, Repetto, Ortiz, and Marsden introduced an improved

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variant of Newmark's method which is energy dissipative at contact, but still unable to avoid the oscillations at contact boundaries. For this reason, Deuflhard, Krause, and Ertel suggested a *contact–stabilized* Newmark method [6, 19] which avoids the unphysical oscillations and is still energy dissipative at contact. This is the time integration scheme of interest in the present paper.

In view of challenging real life problems (e.g., the motion of a human knee, see [19]), an adaptive control of timestep is of crucial importance in order to increase the efficiency of the contact–stabilized Newmark method (called CSN further on). A mesh of equidistant timesteps can not be expected to be adequate for reaching a given accuracy of the approximation of a reasonable computational effort.

The construction of an adaptive timestep control requires a realistic estimation of the consistency error (cf., e.g., the textbook [5]). As a necessary preparatory step, we studied the stability of dynamical contact problems under perturbation of the initial data [16]. For viscoelastic materials, we found a characterization of a class of problems for which a perturbation result can be expected even in the presence of contact. This gave us the idea about a specific norm in function space which has been exploited for the estimation of the consistency error of Newmark methods. In the unconstrained situation, the symmetric Newmark scheme is equivalent to the Störmer-Verlet scheme which is well–known to be second order consistent (see, e.g., [12]). In the constrained situation, we have proven an estimate for the consistency error of the classical Newmark method, the modified Newmark method by Kane et al., and the contact–stabilized Newmark method under the assumption of bounded total variation of the solution [17].

The paper is organized as follows. We will start with a short exposition of the dynamical Signorini contact problem and the contact–stabilized Newmark method in Section 2. Further, we will sum up known consistency and sensitivity results for the scheme. In Section 3, we will analyze the existence of an asymptotic error expansion of the discretization error theoretically as well as numerically. These results are the basis for the application of modified extrapolation methods in order to construct a comparative scheme of higher order. Finally, in Section 4, we will suggest a problem-adapted error estimator and a suitable timestep selection (called ContacX). We will conclude the paper by a numerical example in Section 5.

2 Notation and Background

In order to fix notation, we write down the classical contact problem formulation for linearly viscoelastic materials via Signorini's contact conditions. Afterwards, we present the corresponding contact–stabilized Newmark method, and we review existing sensitivity and consistency results for the scheme.

2.1 Problem formulation

Our model for dynamical contact between two bodies is based on linearized Signorini's contact conditions. In view of existing perturbation and consistency results, see [16] and [17], we consider linear viscoelastic bodies fulfilling the Kelvin-Voigt constitutive law. For the convenience of the reader, here we merely collect the notation used therein.

Notation. Let the two bodies be identified with the union of two domains which are understood to be bounded subsets in $\mathbb{R}^d$ with $d = 2, 3$. Each of the boundaries are assumed to be Lipschitz and decomposed into three disjoint parts: Γ_D, the Dirichlet boundary, Γ_N, the Neumann boundary, and Γ_C, the possible contact boundary. The actual contact boundary is not known in advance, but is assumed to be contained in a compact strict subset of Γ_C. The Dirichlet boundary conditions give rise to $\mathbf{H}^1_D := \{\mathbf{v} \,|\, \mathbf{v} \in \mathbf{H}^1,\ \mathbf{v}|_{\Gamma_D} = 0\}$.

Tensor and vector quantities are written in bold characters, e.g., $\mathbf{v}$. Time derivatives are indicated by dots ($\dot{}$). For the sake of clear arrangement, we use the abbreviation $\bar{\mathbf{v}} = (\mathbf{v}, \dot{\mathbf{v}})$ for a function and its first time derivative.

For given Banach space $\mathbf{V}$ and time interval $t_0 < T < \infty$, let $C([t_0, T], \mathbf{V})$ be the continuous functions $\mathbf{v} : [t_0, T] \to \mathbf{V}$. The space $\mathbf{L}^2(t_0, T; \mathbf{V})$ consists of all measurable functions $\mathbf{v} : (t_0, T) \to \mathbf{V}$ for which $\|\mathbf{v}\|^2_{\mathbf{L}^2(t_0,T;\mathbf{V})} := \int_{t_0}^{T} \|\mathbf{v}(t)\|^2_{\mathbf{V}}\, \mathrm{d}t < \infty$ holds. We identify $\mathbf{L}^2$ with its dual space and obtain the evolution triple $\mathbf{H}^1 \subset \mathbf{L}^2 \subset (\mathbf{H}^1)^*$ where we denote the dual space to $\mathbf{H}^1$ by $(\mathbf{H}^1)^*$. With reference to this evolution triple, the Sobolev space $\mathbf{W}^{1,2}(t_0, T; \mathbf{H}^1, \mathbf{L}^2)$ means the set of all functions $\mathbf{v} \in \mathbf{L}^2(t_0, T; \mathbf{H}^1)$ that have generalized derivatives $\dot{\mathbf{v}} \in \mathbf{L}^2(t_0, T; (\mathbf{H}^1)^*)$, see, e.g., [24].

We will need the (total) variation $\mathrm{TV}(\mathbf{v}, [t_0, T], \mathbf{V})$ of a function $\mathbf{v} : [t_0, T] \to \mathbf{V}$. The set of all functions from $[t_0, T]$ into $\mathbf{V}$ that have bounded variation is denoted by $\mathrm{BV}([t_0, T], \mathbf{V})$, compare, e.g., [22].

Non-penetration condition. At the contact interface Γ_C, the two bodies may come into contact but must not penetrate each other. We assume a bijective mapping $\phi : \Gamma^S_C \longrightarrow \Gamma^M_C$ between the two possible contact surfaces to be given. Following [8], we define linearized non-penetration with respect to ϕ by

$$[\mathbf{u} \cdot \boldsymbol{\nu}]_\phi(x, t) = \mathbf{u}^S(x, t) \cdot \boldsymbol{\nu}_\phi(x) - \mathbf{u}^M(\phi(x), t) \cdot \boldsymbol{\nu}_\phi(x) \leq g(x)\,, \quad x \in \Gamma^S_C\,.$$

This condition is given with respect to the initial gap

$$\Gamma^S_C \ni x \mapsto g(x) = |x - \phi(x)| \in \mathbb{R}$$

between the two bodies in the reference configuration, and we have set

$$\boldsymbol{\nu}_\phi = \begin{cases} \dfrac{\phi(x) - x}{|\phi(x) - x|}\,, & \text{if} \quad x \neq \phi(x)\,, \\ \boldsymbol{\mu}^S(x) = -\boldsymbol{\mu}^M(x)\,, & \text{if} \quad x = \phi(x)\,. \end{cases}$$