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A 0-1 Program for Minimum Clustering in Downlink Base Station Cooperation

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A 0-1 Program for Minimum Clustering in Downlink Base Station Cooperation

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Abstract

Base station cooperation in the downlink of cellular systems has been recently suggested as a promising concept towards a better exploitation of the communication system physical resources. It may offer a high gain in capacity through interference mitigation. This however, comes at a cost of high information exchange between cooperating entities and a high computational burden. Clustering of base stations into subgroups is an alternative to guarantee such cooperation benefits in a lower scale. The optimal definition of clusters, however, and a systematic way to find a solution to such problem is not yet available. In this work, we highlight the combinatorial nature of the problem, exploit this to describe the system of users and base stations as a graph and formulate a pure 0-1 program. Its solution suggests a cost optimal way to form clusters and assign user subsets to them.

I. INTRODUCTION

An important issue in modern wireless communications is to develop techniques that mitigate co-channel intercell interference. This constitutes a major problem in the effort to optimally exploit the available physical resources such as frequency spectrum, time and energy. In the recent years, it has been analytically shown that cooperative transmission between base stations (BSs) of neighbouring cells can offer a high capacity gain [1], [2]. The costs of such a communications scenario approach are related to backhaul connections between the cooperating entities, increased signaling for information exchange and high computational effort.

The concept behind the performance optimization is that the entire system, including all base stations and user terminals, can be seen as a virtual (or network [3]) MIMO system and - provided the necessary information exchange - the transmission of all BSs can be *coordinated* in a way such that interference is minimized. An example of such an optimal cooperation can be to choose the precoding matrix in the downlink of all users in the system as a pseudo-inverse prefilter, also known as the Zero-Forcing precoder [1]. Such a choice results in interference free signal reception at all user ends. Alternative ways to chose the precoding matrix is by combining Zero-Forcing with Dirty Paper coding [1]. Such results however can be reached only through a huge information exchange, involving the estimated values of the channel coefficients between all users and all system BSs and further costs related to backhaul connections and channel bandwidth reservation.

Usually, the benefits of BS cooperation are considerable even in smaller subsets of the system BSs, which constitute *clusters*. In such clusters the required information available is reduced. On the other hand, users will still suffer inter-cluster interference. Suggestions of such limited cooperation can be found in [3], [4] and [5]. Clusters can be formed statically or dynamically and certain suggestions are found in the literature for both such approaches [6], [4], [7], [8], [9].

Since each user in a cluster can be served only by the base station subset that defines it, all entries of its precoding vector, related to base stations outside, should be set to zero, as shown in [9]. This leads to the conclusion, that the problem of optimal cluster formation is of combinatorial nature. The optimal user assignment to base stations should define which base stations form the serving clusters. On the other hand, it is important - due to cooperation costs - to keep the size of clusters as small as possible.

The current work is based on the above observation, in order to formulate and solve an exact 0-1 program, which defines the minimum cost BS clusters for cooperation within the cellular network. To do this, the global information over the user channel long term fading coefficients should be available at a central unit, where the optimization is considered to be performed. The importance of our contribution lies in the originality of the formulated optimization problem, as well as in the presentation of a systematic way to define clusters and treat problems of optimal clustering within the physical transmission framework.

The remainder of the paper is organized as follows. The general transmission scheme in the downlink of a cooperating virtual MIMO system is presented in section II, where the influence of clustering at the received SINR of the users is discussed. Section III provides a discription of the system as a graph and assignment variables, cooperation variables, cooperation scenarios and clusters are formally defined. Section IV begins with a statement of the optimization objective and introduces the assignment variables in the beamforming vector. A set of constraints for the problem is given, so that the outcome of the solution is well defined. Properties of the feasible and optimal solutions are presented. In section V, the solution software and results for example instances are provided. Section VI concludes our work.

II. MULTICELL DOWNLINK TRANSMISSION

We consider a set of users $\mathcal{V}^U : |\mathcal{V}^U| = N$, having a fixed position with respect to a set $\mathcal{V}^B : |\mathcal{V}^B| = M$ of single antenna base stations (BSs) throughout the optimization period. The signal vector to be transmitted is given by the $N \times 1$ complex-valued vector $\mathbf{s} = [s_1, \ldots, s_N]^T$, $s_u \in \mathbb{C}$. The user signals are considered independent realizations of a random process with a certain probability distribution. The expected power of each user signal equals p_u, whereas the signals of different users are uncorrelated, so that $\mathbb{E}\left[s_u \cdot s_u^*\right] = p_u$ and $\mathbb{E}\left[s_u \cdot s_n^*\right] = 0$, $n \neq u$.

Following [1], the set of BSs and users forms a generalized Multiple Input Multiple Output (MIMO) system, which implies that each user can potentially be served by each BS. The geographically remote BSs form altogether a virtual antenna array, which communicates with the user virtual array.

Each user's signal is mapped to BSs using a so called *beamforming or precoding vector* $\mathbf{w}_u := [w_{u,1} \ldots, w_{u,M}]^T$ with dimension $M \times 1$. The elements of such a vector are considered again complex numbers $w_{u,b} \in \mathbb{C}$.

After this mapping, the $M \times 1$ antenna signal vector $\mathbf{x}$ for transmission in the downlink is formed. For this, the signal vector is multiplied by the $M \times N$ precoding matrix $\mathbf{W} := [\mathbf{w}_1, \ldots, \mathbf{w}_N]$, that is

$$\mathbf{x} = \mathbf{W} \cdot \mathbf{s}$$

The power of the transmitted signal per antenna can be calculated as follows

$$\mathbb{E}\left[\mathbf{W}\cdot\mathbf{s}\cdot(\mathbf{W}\cdot\mathbf{s})^{\dagger}\right] \;=\; \mathbf{W}\begin{bmatrix} p_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & p_N \end{bmatrix}\mathbf{W}^{\dagger}$$

where $\mathbf{W}^{\dagger}$ is the complex conjugate transpose of $\mathbf{W}$.

The per antenna power expenditure, which - from the above - equals

$$\sum_{u} |w_{u,b}|^2 \cdot p_u \tag{1}$$

describes the consumption of the transmission power physical resource of the system, for each b of the M BSs.

The complex signal x_b transmitted by each antenna to the serviced users experiences fading, with magnitude that depends on the user-BS distance and the stochastic behavior of the channel. It is assumed that each user $u \in \mathcal{V}^U$ has the ability to estimate the channel fading coefficient between itself and each of the BSs, thus forming a complex vector $\mathbf{h}_u := [h_{u,1}\ldots, h_{u,M}]$ of size $1\times M$, with elements $h_{u,b}\in\mathbb{C}$. The $N\times M$ channel matrix is further denoted by $\mathbf{H} := \left[\mathbf{h}_1^T,\ldots,\mathbf{h}_N^T\right]^T$. In TDD systems e.g. the channel estimation can be done by using pilot symbols in the uplink and - assuming reciprocity of the uplink-downlink channel - each BS can inform the user of its current fading value.

The signals received in the downlink by the N users equal

$$\mathbf{y} = \mathbf{H}\cdot\mathbf{W}\cdot\mathbf{s} + \eta \tag{2}$$

where $\mathbf{y} := [y_1,\ldots,y_N]^T$ is the N-dimensional receive signal column vector and $\eta := [\eta_1,\ldots,\eta_N]^T$ is the N-dimensional noise column vector, with $\eta_u\in\mathbb{C}$ zero-mean additive Gaussian noise at user's u receiver end with variance $\mathbb{E}\left[\eta_u\cdot\eta_u^*\right] = \sigma_u^2$ and $\mathbb{E}\left[\eta_u\cdot\eta_n^*\right] = 0$, $n\neq u$. The per user received signal equals

$$y_u \;=\; \mathbf{h}_u\cdot(\mathbf{w}_1 s_1 + \ldots + \mathbf{w}_N s_N) + \eta_u \tag{3}$$

To calculate the received power at u we take expectation over transmitted signals and noise. Due to independence of signal and noise random realizations of different users

$$\mathbb{E}\left[\|y_u\|_2^2\right] \;=\; \mathbf{w}_u^{\dagger}\mathcal{R}_u\mathbf{w}_u\cdot p_u + \sum_{n\neq u}\mathbf{w}_n^{\dagger}\mathcal{R}_u\mathbf{w}_n\cdot p_n + \sigma_u^2$$

where

$$\mathcal{R}_u \;:=\; \mathbf{h}_u^{\dagger}\cdot\mathbf{h}_u. \tag{4}$$

When interference is treated as noise, the Signal-to-Interference-Noise Ratio (SINR) for each user u is

$$SINR_u := \frac{\mathbf{w}_u^{\dagger}\mathcal{R}_u\mathbf{w}_u\cdot p_u}{\sum_{n\neq u}\mathbf{w}_n^{\dagger}\mathcal{R}_u\mathbf{w}_n\cdot p_n + \sigma_u^2} \geq \gamma_u \tag{5}$$

and in order for a level of Quality-of-Service (QoS) per user to be guaranteed, this should be above a predefined threshold, which depends on its receiver and the transmission modus.

In the above formulation, one can observe that each user $n \neq u$, depending on its assignment to the BSs by the beamforming vector, contributes $\mathbf{w}_n^{\dagger} \mathcal{R}_u \mathbf{w}_n \cdot p_n$ to the interference for user u. This term is written more clearly as

$$\mathbf{w}_n^{\dagger} \mathcal{R}_u \mathbf{w}_n \cdot p_n = \left(\sum_{b_i} \sum_{b_j} w_{n,b_i}^* h_{u,b_i}^* h_{u,b_j} w_{n,b_j} \right) p_n \tag{6}$$

When choosing the Zero-Forcing precoder

$$\mathbf{W} \;=\; \mathbf{H}^{\dagger} \left(\mathbf{H} \mathbf{H}^{\dagger} \right)^{-1}$$

it is possible for the user signals to be received interference free $y_u = s_u + \eta_u$, as implied by a simple substitution in (2). Such a choice however is prohibitive in systems with a large number of BSs, since the above precoding strategy would require an enormous amount of data exchange and computational effort due to the problem dimensioning, as well as frequent updates considering that the entries $h_{u,b}$ refer to fast fading coefficients.

These drawbacks can be partly mitigated by grouping the BSs into clusters that serve a specific subgroup of the user set. Such an approach can still provide the benefits of cooperative techniques and interference mitigation in a smaller scale and most importantly with lower costs. In such a case, interference can be avoided within the cluster and the users suffer only inter-cluster interference. To our best knowledge, how clusters should optimally be chosen within the network is not yet clear from the available literature. After the clusters are defined, zero-forcing or other types of precoders can be applied within the cluster BS subset.

Clusters should be formed based on long term channel fading coefficients, since the cooperation between base stations due to protocol signaling, requires a certain time interval to be established and should not be changed on the scale of instantaneous channel measurements. Considering long term measurements, the random effects of fast fading can be averaged out and the matrix $\mathcal{R}$ can be approximated by the channel covariance matrix for user u

$$\tilde{\mathcal{R}}_u := \; \mathbb{E}\left[\mathbf{h}_u^{\dagger} \cdot \mathbf{h}_u \right] \tag{7}$$

which is diagonal, since we consider the case of single antenna BSs and the channels from different BSs to the same terminal u are independent realizations of some random fading process, in other words $\mathbb{E}\left[h_{u,b_i}^* \cdot h_{u,b_j} \right] = 0$. In LTE systems, user terminals have the ability to gather so called RSRP measurements [10] over the instantaneous channel power, which can be averaged over a certain time window to get an unbiased estimator of the channel power expectation. Further information over the channel fading angle is not any more required in such case.

$$g_{u,b}^2 \approx \mathbb{E}\left[|h_{u,b_i}|^2 \right]. \tag{8}$$

By replacing $\mathcal{R}_u$ by (7) in the SINR, we get the following simplification

$$\mathbf{w}_n^{\dagger} \tilde{\mathcal{R}}_u \mathbf{w}_n \cdot p_n = \left(\sum_{b} |w_{n,b}|^2 \cdot g_{u,b}^2 \right) p_n. \tag{9}$$

In what follows, we first provide a graph description of the problem, and further formulate it - based on the scenario described above - as a 0-1 program, having as variables the cooperation between BSs and the assignment of users to clusters.