TECHNISCHE UNIVERSITÄT CHEMNITZ

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Abstract

We give an alternate description of Koiter's shell equation that does not depend on the special mid surface coordinates, but uses differential operators defined on the mid surface.

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1 Introduction

We consider the deformation of a thin shell of constant thickness h under mechanical loads.

If a usual linear elastic material behavior is proposed, then consequently a linearized strain tensor has to be considered. In this case, the well established Koiter shell equation is obtained after some additional simplifications.

We consider these simplifications from the initial large strain equation to the Koiter shell equation in an easier form due to Ciarlet [2]. Based on this, we are able to find a coordinate free description, which means that differential operators (defined on the mid surface of the shell) are used instead of derivatives with respect to the surface parameters (coordinates) (η^1, η^2).

2 Basic differential geometry

2.1 The initial mid surface

We start with the description of the basic differential geometry on both the undeformed shell (initial domain) and the shell after deformation. All vectors and matrices belonging to the initial configuration (mainly the co- and contravariant basis vectors and the matrices of first and second fundamental forms) are written as capital letters. All these quantities belonging to the deformed structure are the same lower case letters. Let

$$\mathcal{S}_0 = \left\{ \boldsymbol{Y}\left(\eta^1, \eta^2\right) : \left(\eta^1, \eta^2\right) \in \Omega \subset \mathbb{R}^2 \right\}$$

be the mid surface of the undeformed shell, where $\boldsymbol{Y}$ denote the points of the surface in the 3-dimensional space and (η^1, η^2) run through a parameter domain Ω. Then we have

$$\begin{aligned} \boldsymbol{A}_i &= \frac{\partial}{\partial \eta^i} \boldsymbol{Y} \quad \text{the tangential vectors } i = 1, 2 \\ \boldsymbol{A}_3 &= \boldsymbol{A}^3 = (\boldsymbol{A}_1 \times \boldsymbol{A}_2) \,/\, |\boldsymbol{A}_1 \times \boldsymbol{A}_2| \quad \text{surface normal vector.} \end{aligned}$$

This defines the first metrical fundamental forms $A_{ij} = \boldsymbol{A}_i \cdot \boldsymbol{A}_j$ written as the (2×2)-matrix

$$\underline{A} = (A_{ij})_{ij=1}^2 .$$

The surface element is

$$d\mathcal{S} = |\boldsymbol{A}_1 \times \boldsymbol{A}_2| \; d\eta^1 d\eta^2 = (det\underline{A})^{1/2} \; d\eta^1 d\eta^2$$