TECHNISCHE UNIVERSITÄT CHEMNITZ

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Abstract

The main focus of this work lies on the simulation of the deformation of mechanical components that consist of nonlinear elastic, incompressible material and that are subject to large deformations. Starting from a nonlinear formulation a discrete problem can be derived by using linearisation techniques and an adaptive mixed finite element method. It turns out to be a saddle point problem that can be solved via a Bramble-Pasciak conjugate gradient method. With some modifications the simulation can be improved.

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1 Introduction

1.1 Content

The object of this work is the numerical simulation of mechanical components, that consist of nonlinear elastic and (nearly) incompressible material and which are subject to a large deformation. As a special case that also includes linear elastic material behaviour with small deformations. The special property of incompressible material, namely the constant volume during any shape changing deformation, needs special treatment in the mathematical formulation. Therefore a mixed ansatz plays an important role.

1.2 Notation

In order to describe the considered problem of deformation, we need several mechanical quantities and the corresponding operators. These operators mostly are defined as tensors of order n and the space of these tensors is denoted with $\mathbb{T}_n$. By choosing a fixed basis the *notation of Voigt* can be used. That allows the representation of the tensors as matrices or vectors. For distinction we use different typefaces for different types of values. This is shown in table 1.1, only few exceptions can occur.

$Q,\ \phi$	scalar function, tensor of order zero
$\boldsymbol{V},\ \boldsymbol{v}$	vector field, tensor of order one
$\boldsymbol{\mathcal{T}}, \boldsymbol{\sigma}$	tensor of order two
$\mathfrak{M}$	tensor of order four
$\underline{a},\ \underline{R}$	n-vector of expanding coefficients
A	matrix

Table 1.1: types of notation

Throughout this paper, pairs of vectors, such as $\boldsymbol{UV}$, form a 2^{nd} order tensor and in general a 2^{nd} order tensor is any linear combination of such pairs. In the same way, a pair of 2^{nd} order tensors, such as $\boldsymbol{\mathcal{E}\mathcal{F}}$, defines a 4^{th} order tensor. Obviously any 2^{nd} order tensor $\boldsymbol{\mathcal{E}}$ is a linear operator with $\boldsymbol{\mathcal{E}}:\ \mathbb{T}_1 \to \mathbb{T}_1$, as well as any 4^{th} order tensor $\mathfrak{M}$ is a linear operator with $\mathfrak{M}:\ \mathbb{T}_2 \to \mathbb{T}_2$.

As usual we define a dot product $\boldsymbol{V} \cdot \boldsymbol{U} \in \mathbb{R}$ for all first order tensors $\boldsymbol{U},\ \boldsymbol{V} \in \mathbb{T}_1$.