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Abstract

This paper presents three new coupling methods for interior penalty discontinuous Galerkin finite element methods and boundary element methods. The new methods allow one to use discontinuous basis functions on the interface between the subdomains represented by the finite element and boundary element methods. This feature is particularly important when discontinuous Galerkin finite element methods are used. Error and stability analysis is presented for some of the methods. Numerical examples suggest that all three methods exhibit very similar convergence properties, consistent with available theoretical results.

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1. Introduction

This paper is concerned with coupling methods for finite element and boundary element methods. Such coupling methods are advantageous for problems whose domain involves an interior finite subdomain(s) embedded in an exterior unbounded subdomain, such that in the interior subdomain the governing partial differential equations are complex and require finite element methods, whereas in the exterior subdomain the governing partial differential equations are simple and can be solved using boundary element methods. The coupling methods are well established in the literature for classical (continuous) finite element and boundary element methods; we refer to [11] and the references given therein.

This paper is motivated by applications that require discontinuous Galerkin (DG) rather than continuous finite element methods. Coupling methods involving DG finite element methods have been analyzed by Bustinza, Gatica, Heuer, and Sayas [2, 3, 8, 9], who established that essentially any boundary element method can be combined with any DG finite element method, as long as one uses approximations continuous on the interface. For two–dimensional problems, this restriction can be removed if one combines DG finite element methods with a particular Galerkin boundary element method [8].

The coupling methods considered by Bustinza, Gatica, Heuer, and Sayas are based on the symmetric formulation of boundary integral equations. In this case, unique solvability of the coupling method is a direct consequence of the unique solvability of the underlying finite element and boundary element systems. A disadvantage of symmetric boundary element methods is that they involve the hypersingular boundary integral operator that not only precludes the use of basic collocation schemes but also requires functions continuous on the interface. The latter restriction is particularly undesirable for coupling methods involving DG finite element methods in $\mathbb{R}^3$ [8].

In this paper, we present three new methods that allow for discontinuous functions on the interface, and therefore significantly simplify the coupling between DG finite element methods with either Galerkin or collocation boundary element methods. The first method is based on the Johnson–Nédélec coupling [13] extended to DG finite element methods. This method admits both collocation and Galerkin boundary element methods. However, the method gives rise to non–symmetric linear algebraic problems and its mathematical foundations have not been established. The second method, which combines a three–field approach [1] and a symmetric boundary integral formulation, addresses some of the drawbacks of the first method, but it involves the hypersingular operator, and therefore it is limited to Galerkin boundary element methods. This method gives rise to non–symmetric but well–structured linear algebraic problems that can be solved almost as efficiently as symmetric ones. Following the coupling approach of DG and mixed finite element methods [10] the third method gives one two options. The first option admits both collocation and Galerkin schemes and results in non–symmetric linear algebraic problems. This option has the advantage of having a sound mathematical foundation for the Galerkin scheme. The second option is limited to Galerkin boundary element methods, but it results in symmetric