TECHNISCHE UNIVERSITÄT CHEMNITZ

Jens Rückert Arnd Meyer

Kirchhoff Plates and Large Deformation

CSC/12-01

Chemnitz Scientific Computing
Preprints

Impressum:

Chemnitz Scientific Computing Preprints — ISSN 1864-0087
(1995–2005: Preprintreihe des Chemnitzer SFB393)

Herausgeber:
Professuren für
Numerische und Angewandte Mathematik
an der Fakultät für Mathematik
der Technischen Universität Chemnitz

Postanschrift:
TU Chemnitz, Fakultät für Mathematik
09107 Chemnitz

Sitz:
Reichenhainer Str. 41, 09126 Chemnitz

http://www.tu-chemnitz.de/mathematik/csc/

TECHNISCHE UNIVERSITÄT CHEMNITZ

Chemnitz Scientific Computing Preprints

Jens Rückert Arnd Meyer

Kirchhoff Plates and Large Deformation

CSC/12-01

CSC/12-01 ISSN 1864-0087 April 2012

Contents

Author's addresses:

Jens Rückert
Arnd Meyer
TU Chemnitz
Fakultät für Mathematik
D-09107 Chemnitz

http://www.tu-chemnitz.de/mathematik/

1 Introduction

In the simulation of deformations of plates it is well known that we have to use a special treatment of the thickness dependence. Therewith we achieve a reduction of dimension from $3D$ to $2D$.

For linear elasticity and small deformations several techniques are well established to handle the reduction of dimension and achieve acceptable numerical results. In the case of large deformations of plates with non-linear material behaviour there exist different problems. For example the analytical integration over the thickness of the plate is not possible due to the non-linearities arising from the material law and the large deformations themselves. There are several possibilities to introduce a hypothesis for the treatment of the plate thickness from the strong Kirchhoff assumption on one hand up to some hierarchical approaches on the other hand.

In this preprint we consider a model of using the Kirchhoff assumption. So first we give a short overview in useful differential geometry and explain the hypothesis mathematically.

The following section treats with the approximate solution of the PDE, we concern with. The preprint ends with some numerical examples.

2 The $3D$-deformation energy

In the entire paper we use the Einstein summation convention. Furthermore we understand a second order tensor as a pair of two vectors, for example $\boldsymbol{A}^1\boldsymbol{A}^2$ or similar. In general, it is any linear combination of such pairs. Accordingly tensors of higher order are built in the same manner. By the way we can consider a vector as a first order tensor.
Some calculation rules are defined for second order tensors. So the dot product maps the (3-dimensional) vector functions $\boldsymbol{U}$ onto vector functions again:

$$(\boldsymbol{A}^1\boldsymbol{A}^2)\cdot\boldsymbol{U} = \boldsymbol{A}^1(\boldsymbol{A}^2\cdot\boldsymbol{U})$$
$$\boldsymbol{U}\cdot(\boldsymbol{A}^1\boldsymbol{A}^2) = \boldsymbol{A}^2(\boldsymbol{A}^1\cdot\boldsymbol{U}).$$

The trace is defined as $tr(\boldsymbol{A}^1\boldsymbol{A}^2) = \boldsymbol{A}^1\cdot\boldsymbol{A}^2$ and the transposed tensor as $(\boldsymbol{A}^1\boldsymbol{A}^2)^\tau = \boldsymbol{A}^2\boldsymbol{A}^1$. The double dot product between two second order tensors is a scalar function. It is defined by

$$\boldsymbol{A}^1\boldsymbol{A}^2 : \boldsymbol{A}^3\boldsymbol{A}^4 = (\boldsymbol{A}^2\cdot\boldsymbol{A}^3)(\boldsymbol{A}^1\cdot\boldsymbol{A}^4).$$

Throughout this paper we display $2nd$ order tensors by gently rolled capital letters. Vectors are written in bold and matrices in underlined types. Therefor we