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Steiner Tree Packing Revisited

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Steiner Tree Packing Revisited

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Abstract

The Steiner tree packing problem (STPP) in graphs is a long studied problem in combinatorial optimization. In contrast to many other problems, where there have been tremendous advances in practical problem solving, STPP remains very difficult. Most heuristics schemes are ineffective and even finding feasible solutions is already NP-hard. What makes this problem special is that in order to reach the overall optimal solution non-optimal solutions to the underlying NP-hard Steiner tree problems must be used. Any non-global approach to the STPP is likely to fail. Integer programming is currently the best approach for computing optimal solutions. In this paper we review some "classical" STPP instances which model the underlying real world application only in a reduced form. Through improved modelling, including some new cutting planes, and by emplyoing recent advances in solver technology we are for the first time able to solve those instances in the original 3D grid graphs to optimimality.

1 Introduction

The weighted Steiner tree problem in graphs (STP) can be stated as follows:

> *Given a weighted graph $G = (V, E, c)$ and a non-empty set of vertices $T \subseteq V$ called* terminals, *find an edge set S^* such that $(V(S^*), S^*)$ is a tree of minimal weight that spans T.*

An extensive survey on the state-of-the-art of modeling and solving the STP can be found in [25]. Many papers on the STP claim real-world applications, especially in VLSI-design and wire-routing. This usually refers to a generalization of the STP, the weighted Steiner tree packing problem in graphs (STPP). Instead of having one set of terminals, we have N non-empty disjoint sets $T_1, \ldots, T_N$, called *Nets*, that have to be "packed" into the graph simultaneously, i. e., the resulting edge sets $S_1, \ldots, S_N$ have to be disjoint. In these applications, G is usually some sort of 3D grid graph. [13, 21, 14] give detailed explanations of the modeling requirements in VLSI-design. Three routing models for 2D or 3D grid graphs are of particular interest:

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Channel routing: Here a complete rectangular grid graph is used. The terminals of the nets are exclusively located on two opposing borders. The size of the routing area is not fixed in advance. All nets have only two terminals, i. e., $|T_i| = 2$.

Switchbox routing: We are given a complete rectangular grid graph. The terminals may be located on all four sides of the graph. Thus, the size of the routing area is fixed.

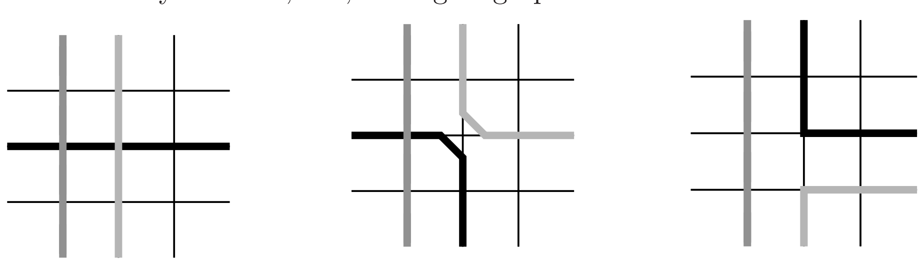
General routing: In this case the grid graph may contain holes or have a non rectangular shape. The size of the routing area is fixed and the terminals may be located arbitrarily.

The intersection of the nets is an important issue in Steiner tree packing. Again three different models are possible:

Manhattan (Fig 1(a)) Consider some (planar) grid graph. The nets must be routed in an edge disjoint fashion with the additional restriction that nets that meet at some node are not allowed to bend at this node, i. e., so-called *Knock-knees* are not allowed. This restriction guarantees that the resulting routing can be laid out on two layers at the possible expense of causing long detours.

Knock-knee (Fig 1(b)) Again, some (planar) grid graph is given and the task is to find an edge disjoint routing of the nets. In this model Knock-knees are possible. Very frequently, the wiring length of a solution is smaller than in the Manhattan model. The main drawback is that the assignment to layers is neglected.

Node disjoint (Fig 1(c)) The nets have to be routed in a node disjoint fashion. Since no crossing of nets is possible in a planar grid graph, this requires a multi-layer model, i. e., a 3D grid graph.



(a) Manhattan model (b) Knock-knee model (c) Node disjoint model

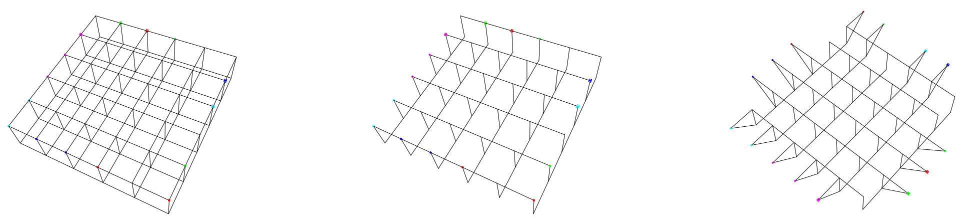
Figure 1: STPP intersection models

While channel routing usually involves only a single layer, switchbox and general routing problems are typically multi-layer problems. Using the Manhattan and Knock-knee intersection is a way to reduce the problems to a single layer. Accordingly, the multi-layer models typically use node disjoint intersection. While the multi-layer model is well suited to reflect reality, the resulting graphs become quite large. We consider two[1] possibilities to model multiple layers:

1 A third possibility is to use a single-layer model with edge capacities greater than one.

k-crossed layers (Fig 2(a)) There is given a k-dimensional grid graph (i. e., k copies of a grid graph are stacked on top of each other and corresponding nodes are connected by perpendicular lines, so-called *vias*), where k denotes the number of layers. This is called the k-layer model in [21].

k-aligned layers (Fig 2(b)) This model is similar to the crossed-layer model, but in each layer there are only connections in one direction, either east-to-west or north-to-south. [21] calls this the *directional* multi-layer model. [20] indicate that for $k = 2$ this model resembles the technology used in VLSI-wiring best. [4] mentions that current technology can use a much higher number of layers (20 and more).



(a) Multi-crossed layers (b) Multi-aligned layers (c) With connectors

Figure 2: STPP modeling taxonomy

Note that for switchbox routing there is a one-to-one mapping between feasible solutions for the Manhattan one-layer model (MOL) and the node disjoint two-aligned-layer model (TAL), assuming that there are never two terminals on top of each other, i. e., connected by a via.

For the general routing model, this mapping might not be possible. If a terminal is within the grid there is no easy way to decide the correct layer for the terminal in the two-layer model.

Unfortunately, in the seven “classic” instances given by [6, 23, 8] two terminals are connected to a single corner in several cases. This stems from the use of *connectors*, i. e., the terminal is outside the grid and connected to it by a dedicated edge. In the multi-layer models there has to be an edge from the terminal to all permissible layers (Fig 2(c)).

The Knock-knee one-layer model can also be seen as an attempt to approximate the node disjoint two-crossed-layer model. But mapping between these two models is not as easy. [5] have designed an algorithm that guarantees that any solution in the Knock-knee one-layer model can be routed in a node disjoint four-crossed-layer model, but deciding whether three layers are enough has been shown to be $\mathcal{NP}$-complete by [22].

2 Integer programming models

A survey of different integer programming models for Steiner tree packing can be found in [7]. We will examine two of the models in more detail.