Optimum Design of a Cantilevered Unimorph Beam with Piezoelectric Sensors in Vortex Shedding

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Abstract:

A cantilevered piezoelectric generator vibrating in the region of vortex shedding was designed and analyzed electro-mechanically. CFD analyses were performed to obtain vortex shedding frequency around various shapes of bodies placed in an air stream. The optimum topology of a piezoelectric material layer was obtained by a newly developed topology optimization technique for piezoelectric materials which utilized the electromechanical coupling equations, MMA (method of moving asymptotes), and SIMP (solid isotropic material with penalization) interpolation. Using the topology optimization, several cantilevered beam-type piezoelectric generators which fluctuates in vortex shedding has been developed. The voltage output results from topology optimization were validated by experiments.

Key words: Piezoelectric material, topology optimization, voltage, vortex shedding, bimorph electricity generator

Introduction

Piezoelectric materials can produce measurable electricity (piezoelectricity) when their static structure is deformed by about 0.1% of the original dimension. This phenomenon is called as the direct piezoelectric effect that means the internal generation of electrical charge resulting from an applied mechanical force. Piezoelectricity is used for many useful applications such as the detection of forces, voltage generation, and suppressing means of vibrations. The most commonly used piezoelectric materials for sensing and actuating are PZT (lead zirconate titanate), PMN-PZT and PVDF (polyvinylidene difluoride). Even though PZT has been widely used in many engineering applications, it is brittle and therefore the fatigue life is limited by the repeated alternating strain level. Compared to the PZT material, PVDF is more flexible and easier to be formed in a required shape and it can withstand larger amounts of strain.

A new analytical model of bending vibration was developed based on long and slender beams with tip masses and the generation of electricity was investigated by changing beam shapes obtained from the shape optimization process [1]. The topology optimization technique was applied to improve mechanical to electrical energy conversion in a static sense for piezoelectric plates [2]. Rupp *et al* [3] developed a general methodology to analyze and design a piezoelectric electrical energy generator based on multilayer plate and shell structures with piezoelectric layers coupled to an external circuit. Kim *et al* [4] derived two mathematical conditions that the penalty exponents for topology optimization algorithm with a PZT material had to satisfy for stable convergence for one-dimensional problems, and their effectiveness for two-dimensional problems was studied. Research articles on piezoelectrical generators vibrating by a shaker are easily found. However, such devices vibrating from surrounding natural energy resources are rare.

A cantilevered electricity generating beam with a piezoelectric material layer operated in vortex shedding was developed on the basis of electro-mechanical and computational fluid dynamic analyses. In order to obtain the largest output voltage, the external exciting frequency of the cantilevered generator was tuned to the 1st natural frequency of the beam and the piezoelectric material layer on the cantilever was optimized topologically. An efficient analysis approach for prediction of voltage outputs of the beam has been developed based on the finite element method coupled with a piezoelectric behavior. Using the method, the influences of geometric parameters on power generation were investigated and the electric characteristics were evaluated.

Topology Optimization of Piezoelectric Materials

To perform the topology optimization of piezoelectric materials, an electromechanical coupling coefficient (EMCC) was introduced as,

$$k^2 = \frac{1}{\frac{\varepsilon_{33}{}^{s} c_{33}{}^{E}}{e_{33}{}^{2}} + 1} = \frac{1}{\frac{1}{k_a{}^2} + 1} = \frac{k_a{}^2}{1 + k_a{}^2} \tag{1}$$

$$k_a{}^2 = \frac{(\rho^{ne}\bar{e})^2}{(\rho^{nc}\bar{c}^E)^2(\rho^{n\varepsilon}\bar{\varepsilon}^S)^2} = \rho^{2ne-(nc+n\varepsilon)} \frac{\bar{e}}{\bar{c}^E \cdot \bar{\varepsilon}^S} \tag{2}$$

where $c_{33}{}^{E}$ is an elastic stiffness constant, $\varepsilon_{33}{}^{S}$ is permittivity, e_{33} is a piezoelectric coefficient, k_a is EMCC, ρ is an element density, and ne, nc and $n\varepsilon$ are the power index of an interpolation function to be determined for proper topology optimization.

For the structure made up of piezoelectric and other substrate materials, following EMCC expression is usually applied,

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$$k^2 = \frac{f_1^2 - f_2^2}{f_1^2} \qquad (3)$$

where f_2 is a resonant frequency of a closed circuit and f_1 is that of an open circuit. The objective function for topology optimization of piezoelectric material is,

$$f = \alpha \frac{1}{k} + (1-\alpha) \bullet (\lambda_{EV} - \lambda_{ext}) \qquad (4)$$

where λ_{EV} is an eigenvalue of the energy harvester and λ_{ext} is that of external excitation. The first resonant mode was chosen to be 50 – 300 Hz considering the actual range of frequency. A modified MMA (method of moving asymptotes) was developed as an optimization algorithm and the algorithm was coupled with ANSYS to calculate the electric voltage output of the cantilevered device.

Fig. 1 shows the cantilevered electricity-generating beam with a tip mass and piezoelectric initial design domain. Fig. 2 represents the layout of topology-optimized shape of piezoelectric material layer on the substrate beam. The result of topology optimization shows that the PZT in the center region is removed and the materials near edges remain. The PZT area near fixed points of the cantilevered beam is wider than any other edges.

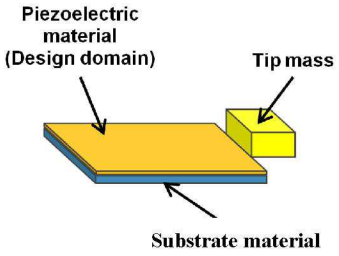

Fig. 1: Cantilevered electricity generating beam with a tip mass and piezoelectric design domain

Fig. 2: The layout of topology-optimized shape of piezoelectric material layer

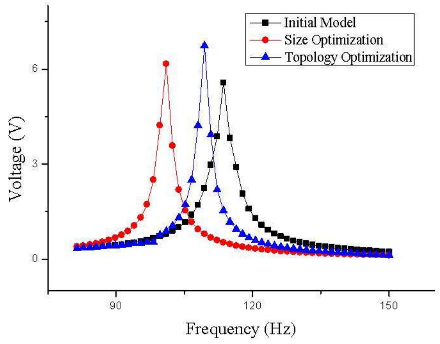

Fig. 3: Comparison between voltage outputs of non-optimized and optimized piezoelectric materials

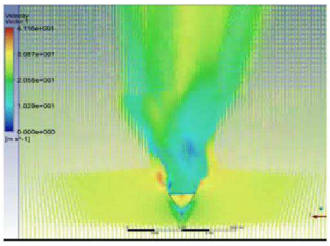

Fig. 4: Occurrence of fluctuating vortex shedding behind an obstacle in an air flow by CFD analysis

Fig. 3 shows the voltage outputs computed by FE analyses for three cases (an initial, a size-optimized and a topology-optimized cantilever) of the same piezoelectric material. The voltage output of the cantilever beam with topology-optimized piezoelectric material is larger than that of any other beams.

Voltage Outputs and Experiments

The required vortex shedding pattern at an air flow speed of 20 m/s was obtained using a flow obstacle by CFD analyses, as in Fig. 4. The good cross-sectional shape of the upstream obstacle which can produce fluctuating vortex at a required frequency of 16 Hz was also calculated in advance. The beam structure in Fig. 2 was placed in the fluctuating vortex stream and the voltage output was calculated by ANSYS finite element analysis with special cares. The several piezoelectric beams were fabricated for validation experiments and put into a wind tunnel with wiring and sensing equipments. As results, the voltage output by FE analysis was obtained as 1.45 V and an average experimental result in the wind tunnel test was 1.2 V. The two results show a very good correlation between FEM analyses and wind tunnel tests.

Conclusion

An efficient analysis and topology optimization method to predict the electric voltage outputs of electric energy generators in vortex shedding has been developed on the basis of the finite element method coupled with piezoelectric behaviors. The voltage output was validated by fabrication the device and wind tunnel tests.

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