Maximizing the Output Power of a Piezoelectric Generator from the Admittance Measurement by selecting the Rectification Technique

Loreto Mateu, Henrik Zessin, Peter Spies
Fraunhofer Institute for Integrated Circuits IIS
Nordostpark 93, Nuremberg, Germany
loreto.mateu@iis.fraunhofer.de

Abstract (style "IWPMA_Conferences_Headline"):

AC-DC converters employed for harvesting power from piezoelectric transducers can be divided into linear (i.e. diode bridge) and non-linear techniques (i.e. synchronized switch harvesting on inductor, SSHI). Depending on the piezoelectric element, a non-linear technique can improve a significant percentage the power harvested. This paper presents an analytical technique based on the measurement of the impedance circle of the piezoelectric element to determine whether the diode bridge harvests a considerable percentage of the available power.

Key words: energy harvesting, rectifiers, piezoelectric transducer, impedance matching, synchronized switch harvesting on inductor.

I. Introduction

AC-DC converters that perform the rectification of the power harvested by piezoelectric transducers are divided in two different groups: non-linear (i.e. parallel SSHI) and linear (i.e. diode bridge). A non-linear technique implies the inversion of the piezoelectric voltage through an inductor when the maximum displacement of the piezoelectric element is reached. The paper presents a method to estimate the electrical impedance of AC-DC converters in order to predict the harvested power to be obtained with a certain piezoelectric transducer from the measurement of its impedance circle.

The paper is divided as follows: Section II describes how to calculate the elements of the piezoelectric equivalent circuit at resonance frequency from the measurement of the impedance circle. In section III, it is calculated the impedance of the mechanical part of the piezoelectric transducer as well as the electrical part when a resistive load is connected. These values are employed to calculate the power ratio that is defined as the power harvested by a resistive load to the maximum harvested power. Section IV presents an analytical method to calculate the electrical impedance of a diode bridge connected to the piezoelectric element done and section V presents the case of the non-linear technique. A summary of the results obtained with two piezoelectric elements is given in section VI. Finally, Section VII summarizes the conclusions.

II. Piezoelectric Equivalent Electrical Impedance in Open Circuit

Measurements of the admittance of different piezoelectric transducers have been done with an AUTOLAB PGSTAT302N potentiostat/galvanostat. Afterwards, the values of the elements that conform the electrical equivalent circuit at resonance frequency, $L^{'}_{n}$, $C^{'}_{n}$, $R^{'}_{n}$ in series and C_p connected in parallel to them, have been calculated. The admittance is calculated from the circuit analysis of the equivalent circuit:

$$Y(s) = C_p s \frac{s^2 + s\frac{R'_n}{L'_n} + \frac{C'_n + C_p}{C_p C'_n L'_n}}{s^2 + s\frac{R'_n}{L'_n} + \frac{1}{L'_n C'_n}} \quad (1)$$

The values of the elements of the piezoelectric transducer equivalent circuit at the secondary side are related to the ones at the primary side by $L'_n \stackrel{\text{def}}{=} \frac{L_n}{N_n^2}$, $R'_n \stackrel{\text{def}}{=} \frac{R_n}{N_n^2}$ and $C'_n \stackrel{\text{def}}{=} C_n N_n^2$.

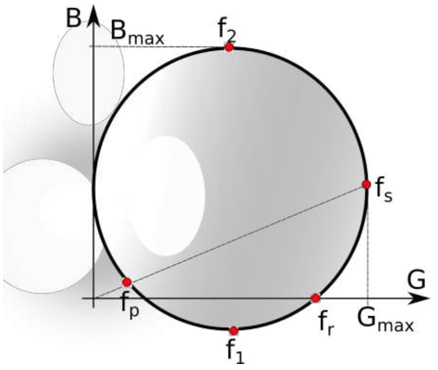

Fig.12: Admittance circle of a piezoelectric element at the resonance frequency.

A fitting curve process is employed to estimate the value of $L^{'}_{n}$, $C^{'}_{n}$, $R^{'}_{n}$ and C_p. ω_r is the resonant frequency that takes place when the susceptance B is null. ω_s corresponds to the series resonance frequency and occurs when the conductance G is maximum. ω_p is defined as the parallel resonance frequency and ω_1 and ω_2 correspond to the frequencies at which the susceptance is minimum and maximum, respectively (see Fig.1) . Thus, the initial value

DOI: http://dx.doi.org/10.2314/IWPMA_67

for C_p employed in the fitting process is calculated using the following equation where the numerator corresponds to the value of the susceptance at the point of the maximum conductance:

$$C_p = \frac{Im(Gmax)}{\omega_s} \tag{2}$$

Combining the two following equations, the values of L'_n and C'_n are determined:

$$\omega_s^2 = \frac{1}{L'_n C'_n} \tag{3}$$

$$C'_n = \frac{1}{\omega_s^2 \omega_2}\left(Im(Bmax)(\omega_s^2 - \omega_2^2) + \frac{C_p(\omega_p^2 - \omega_s^2)}{Re(Gmax)}\omega_2 Re(Bmax) + C_p\omega_2^3\right) - C_p \tag{4}$$

The value of R'_n corresponds to the maximum of the real part of the conductance:

$$R'_n = Re(Gmax) \tag{5}$$

The ratio of the modulus of the maximum conductance to the modulus of the susceptance at the series resonance frequency corresponds to the figure of merit M [1]:

$$M = Q_m K = Q_m k^2 = \frac{|Y_{mo}|}{|Y_d|} = \frac{1/R'_n}{\omega_s C_p} \tag{6}$$

This figures of merit is employed by Guyomar et al.[2] to decide when a non-linear rectifier will harvest more power than a diode bridge. If M ≤ π, the non-linear technique harvests more power than the linear technique.

III. Piezoelectric Equivalent Electrical Impedance with a Load

The analysis done by Brufau-Penella et al. [3] allows to calculate the power ratio that is defined as the ratio of the maximum power harvested with a resistive load to the maximum power harvested with a complex load employing the elements of the piezoelectric equivalent circuit [3]. However, the argumentation related to the Thevenin equivalent model is not valid when a load is connected to the piezoelectric element since in this case the load is placed in parallel to the piezoelectric electrical capacitor C_p as it is exposed in [4].

The electrical equivalent impedance of the mechanical part of the piezoelectric transducer is given by:

$$Z_{mech}(j\omega) = R'_n + j\omega L'_n + \frac{1}{j\omega C'_n} = R_{mech} + jX_{mech} \tag{7}$$

If a resistive load is connected to the piezoelectric transducer, the electrical equivalent impedance corresponds to:

$$Z_{elec}(j\omega) = R_{elec} + jX_{elec} = \frac{R_L}{1+\omega^2 C_p^2 R_L^2}\left(1 - j\omega C_p R_L\right) \tag{8}$$

The harvested power ratio of resistive load to complex load is:

$$P_{ratio} = \frac{4R_{elec\_Phr}R_{mechcomp}}{(R_{mechPhr}+R_{elecPhr})^2+(X_{mechPhr}+X_{elecPhr})^2} \tag{9}$$

Thus, there will be piezoelectric elements for which the power ratio is close to one and others for which is far from one.

IV. Piezoelectric Equivalent Electrical Impedance with a Diode Bridge

The connection of a diode bridge circuit with a resistive load and a filtering capacitor to a piezoelectric generator has two equivalent circuit depending if the diodes are not conducting or are conducting. If the diodes are not conducting, the equivalent circuit is represented in Fig.2 where only the piezoelectric capacitance acts as electrical load.

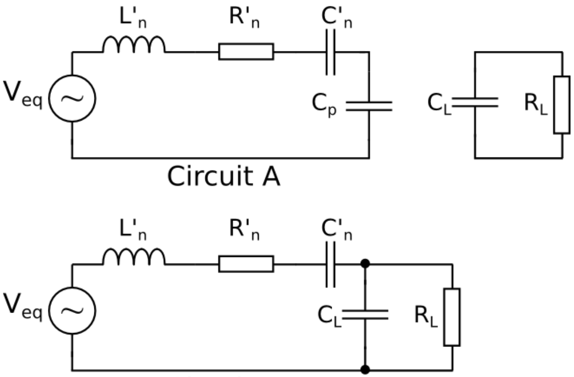

Fig. 2: Equivalent circuit of the piezoelectric transducer while the current is not flowing through the diodes (circuit A) and when it is flowing for the calculation of the electrical impedance.

The transfer function of the electrical impedance for circuit A and B are respectively:

$$Z_{elecdbA}(j\omega) = \frac{1}{j\omega C_p} \tag{10}$$

$$Z_{elecdbB}(j\omega) = \frac{R_L}{1+j\omega R_L C_L} \tag{11}$$

Therefore, the transfer function of the electrical impedance is given by:

$$Z_{elecdb}(j\omega) = \frac{Z_{elecdbA}(j\omega)\left(1-e^{-j\omega\theta}\right)+Z_{elecdbB}(j\omega)\left(e^{-j\omega\theta}-e^{-j\omega T/2}\right)}{1-e^{-j\omega T/2}} \tag{12}$$

where between 0 and θ just the piezoelectric capacitor is connected and between θ and T/2. The transfer function of the electrical impedance has a period T/2. The value of θ is calculated combining the following two equations:

$$V_L = I_L R_L = \frac{1}{T/2}\int_{\theta/\omega}^{T/2}\frac{V_{eq}\omega}{L}\frac{1}{\sqrt{\left(\frac{C_p+C}{LC_pC}-\omega^2\right)^2+\left(\frac{\omega R}{L}\right)^2}}\sin\omega t\, dt \tag{13}$$

$$\cos\theta = 1 - 2\frac{V_L}{V_{oc}} \tag{14}$$

where V_{oc} corresponds to the open circuit voltage of the piezoelectric element.

The power ratio for this case would be:

DOI: http://dx.doi.org/10.2314/IWPMA_67

$$P_{ratio} = \frac{4R_{elecdb}R_{mechcomp}}{(R_{mechdb}+R_{elecdb})^2+(X_{mechdb}+X_{elecdb})^2} \qquad (15)$$

If the power ratio is close to one, then a linear technique like the diode bridge rectifier would give a result as good as a non-linear technique without the complexity at the circuit level that the later has. The harvested power is the power on R_{elecdb} that is not equal to the power on R_L. The power on the load is a fraction of the harvested power.

The results obtained from an electrical simulation of two different piezoelectric transducers connected to a diode bridge have been compared with the results obtained from the equations of this section. It has been noticed that for $|Z_{elecdb}|$ the equations matches the simulations. However, the comparison of $\angle Z_{elecdb}$ carries complications since the determination of the exact time at which it can be considered that the diode is conducting in the electrical simulation is difficult to establish and this introduces an error in the calculation of $\angle Z_{elecdb}$. Moreover, the voltage and power plots obtained from the electrical simulations and the equations matches in form as well as in the point where the maximum power is obtained but the value obtained from the equations is around 20% lower than the value obtained from the electrical simulator. This error is due to the assumption that circuit B does not affect the current flowing through circuit A and that $|I_{eq}|$ is not affected by the load of circuit B.

V. Piezoelectric Equivalent Electrical Impedance with a SSHI Converter

The power ratio of two different piezoelectric elements connected to a parallel SSHI converter has been obtained using an electrical simulation software tool. The values of $\angle Z_{elecSSHI}$ present the same problem that in the electrical simulation of the diode bridge. The power ratio of this topology can be calculated substituting the values obtained for this topology in eq. (15).

A new power ratio is defined as power harvested by the SSHI converter to power harvested by the diode bridge:

$$P_{ratio} = \frac{R_{elecSSHI}\left((R_{mechdb}+R_{elecdb})^2+(X_{mechdb}+X_{elecdb})^2\right)}{R_{elecdb}\left((R_{mechSSHI}+R_{elecSSHI})^2+(X_{mechSSHI}+X_{elecSSHI})^2\right)} \qquad (16)$$

VI. Comparison of the Harvested Power

Tab. 1 and Tab.2 shows the results obtained for two different piezoelectric transducers comparing the four topologies under consideration in terms of harvested power: resistive load, complex conjugate of the equivalent mechanical impedance, diode bridge with a resistor as a load and parallel SSHI converter with a resistor as a load. For the parallel SSHI converter, the values given correspond to the power at the resistive load. The value of M for the piezoelectric element A is 0.42 and for the B is 1.13.

Tab. 1: Piezoelectric element A

	Open circuit	Resistor	Complex conjugate	Diode bridge	Parallel SSHI
f_{Pmax}(Hz)	24.31	24.41	-	24.41	24.38
R_L (kΩ)	-	17.24	-	25.37	41.92
P_{max}(mW)	-	5.28	9.18	5.232	16
R_{mech_Pmax} (kΩ)	38.31	38.31	-	38.31	-
X_{mech_Pmax} (kΩ)	-	5.7	-	5.7	-
R_{elec_Pmax} (kΩ)	-	8.13	-	7.41	-
X_{elec_Pmax} (kΩ)	-16.37	-8.61	-	-11.54	-

Tab. 2: Piezoelectric element B

	Open circuit	Resistor	Complex conjugate	Diode bridge	Parallel SSHI
f_{Pmax}(Hz)	59.18	59.78		59.78	59.38
R_L (kΩ)	-	232.2		393.9	186.9
P_{max}(mW)	-	1.632	1.775	1.632	2.454
R_{mech_Pmax} (kΩ)	198.05	198.05		198.05	-
X_{mech_Pmax} (kΩ)	-	-145.43		-145.43	-
R_{elec_Pmax} (kΩ)	-	113.35		111.01	
X_{elec_Pmax} (kΩ)	224.11	-116.09		-136.21	-

VII. Conclusions

An analytical model that allows calculating the equivalent electrical impedance of a diode bridge connected to a piezoelectric element has been presented. This analysis combined with the identification of the elements of the piezoelectric equivalent circuit allows determining the harvested power. If the harvested power of the diode bridge is close to the harvested power using the complex conjugate of the mechanical impedance as a load, a non-linear technique will not provide a benefit in terms of harvested power.

Further work will include the calculation of the electrical impedance of the parallel SSHI converter to determine the power ratio of this rectifying technique.

References

[1] T. Ikeda, **Fundamentals of Piezoelectricity**, (1990)

[2] D. Guyomar, G. Sebald, S. Pruvost, M. Lallart, A. Khodayari, C. Richard, **Energy Harvesting from Ambient Vibrations and Heat**, Journal of Intelligent Material Systems and Structures , 20 (5), 609-624 (2009)

[3] J. Brufau-Penella, M. Puig-Vidal, **Piezoelectric Energy Harvesting Improvement with Complex Conjugate Impedance Matching**, Journal of Intelligent Material Systems and Structures, 20 (5), 597-608 (2009)

[4] J. Liang, W-H. Liao, **Impedance modeling and analysis for piezoelectric energy harvesting systems**, Mechatronics, IEEE/ASME Transactions on 17.6, 1145-1157 (2012)