

CARLO SCHMITT , LOTHAR WYRWOLL , ALBERT  
MOSER , INCI YUEKSEL-ERGUEN 

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Zuse Institute Berlin  
Takustr. 7  
14195 Berlin  
Germany

Telephone: +49 30 84185-0  
Telefax: +49 30 84185-125

E-mail: [bibliothek@zib.de](mailto:bibliothek@zib.de)  
URL: <http://www.zib.de>

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# Integration of Flexible Distributed Energy Resource Models into a Market Coupling-based Lagrangian Relaxation of the pan-European Electricity Market

Carlo Schmitt<sup>1</sup>, Lothar Wyrwoll<sup>1</sup>, Albert Moser<sup>1</sup>, and Inci Yueksel-Erguen<sup>2</sup>

<sup>1</sup>*Institute for High Voltage Equipment and Grids, Digitalization and Energy Economics (IAEW), RWTH Aachen University, c.schmitt@iaew.rwth-aachen.de; l.wyrwoll@iaew.rwth-aachen.de; info@iaew.rwth-aachen.de*

<sup>2</sup>*Zuse Institute Berlin, yueksel-erguen@zib.de*

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## Abstract

This report presents a method to integrate highly flexible technology models for distributed energy resources such as electric vehicles, power-to-heat systems, or home battery systems into a Lagrangian relaxation of the pan-European day-ahead electricity market (EULR). These flexible technology models are highly sensitive to the changes of Lagrangian multipliers within the iterative Lagrangian relaxation process, leading to volatile behavior. Furthermore, they show a high concurrency in their market behavior due to their technical homogeneity. Therefore, the method proposed in this report is an extension of the existing EULR modeling approach to improve the model's convergence. The methodological extension comprises a convex combination of iteration solutions for the Lagrangian relaxation subproblems similar to Dantzig-Wolfe decomposition. An exemplary case study shows the effectiveness of this extended approach.

**Keywords**— Electricity Market Simulation, Lagrangian Relaxation, Distributed Energy Resources

## 1 Introduction

Following the liberalization of the European energy system, the dispatch of power plants is mainly determined by the coupled wholesale electricity market (either by direct participation or by the effect of market prices on make-or-buy decisions of power plant operators). In that context, the day-ahead market is regarded as the leading market determining base prices and power plant dispatches. Assuming perfect competition, electricity markets can be approximated by cost-minimizing system models [1]. Cost-minimizing unit commitment models considering detailed start-up and shut-down decisions of power plants are an established approach for modeling the European (day-ahead) wholesale electricity market, especially in the energy system planning and

analysis context [2]. The resulting unit commitment optimization problem suffers from high computational complexity due to integer-based on/off decisions of power plants and the complicating market coupling constraints according to the Single Day-ahead Coupling (SDAC) of the different market areas. Thus, solving the detailed European energy system models requires facilitation of decomposition methods or relaxation of the complicating market balance and market-coupling constraints. Lagrangian Relaxation (LR) is preferred to decomposition approaches such as Benders Decomposition [4, 5, 6] or Branch-and-Price [7] in the electricity markets modeling context as it poses several superiorities compared to other decomposition methods. Interpretation of the Lagrangian multipliers of the relaxed market coupling constraints as market prices, which are among significant evaluation criteria of market models, is one of these superiorities. Besides, the LR allows for the formulation of individual subproblems for each thermal power plant, (hydro-) storage system, and renewable generation plants. We can then interpret subproblems as contribution margin optimizations of individual actors (e.g., power plant operators) subject to the given market prices (multipliers) of the LR within a welfare optimization model of the pan-European electricity market. The European Lagrangian Relaxation (EULR) model developed in [3] introduced the consideration of market coupling as part of the LR. The advantage of the EULR model compared to three-stage approaches where the market coupling is determined by a pre-processing step has been shown in [2]. In three-stage approaches (e.g. [8]), first a linear economic dispatch model is solved to determine exchanges between market zones. Secondly, a detailed mixed-integer unit commitment model is solved per market area to determine the detailed power plant schedules including start-ups and shut-downs. This detailed unit commitment is often solved by LR. Third, the European economic dispatch is re-solved with the fixed binary decision variables. In contrast to sequential implementation of three-step approaches, the EULR model integrates these steps. A LR of European market balancing and coupling constraints is carried out. The Lagrangian multipliers are interpreted as electricity prices per market area. An additional market coupling subproblem determines the optimal electricity exchange between market areas as well as the accepted schedules of each power plant. The overdemands and oversupplies in each market area resulting from the market coupling subproblem are then used as subgradients for the adjustment process of the Lagrangian multipliers. The update process further takes export and import potentials between market areas into account to ensure price convergence between market areas [3]. An overview of the EULR approach is given in Figure 1.

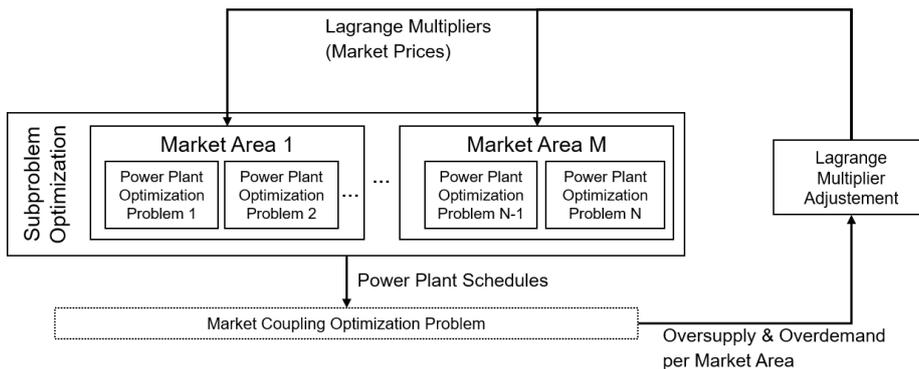


Figure 1: Overview on EULR [3] process

## 1.1 Problem definition

The EULR approach can further be extended to take into account distributed energy resources (DERs) such as battery storage systems and heating technologies (heating pumps, distributed combined-heat-and-power, heat storage systems) as done in [9]. The market integration of DERs increases the available flexibility potential, thus potentially reducing system costs and improving integration of variable renewable generation. In this report, DER (sub-)problems are used synonymously for different kinds of technology (sub-)models such as battery storage systems, electric vehicles and vehicle fleets, and heating systems in combination with heating storage systems. Furthermore, also virtual power plants combining those technologies are represented by those DER (sub-)problems. However, the DER technologies integration into the iterative LR approach might lead to some convergence problems. We illustrate these problems with the help of the following example which is also depicted in Figure 2.

Let us assume an electricity system with two power plants with maximum power

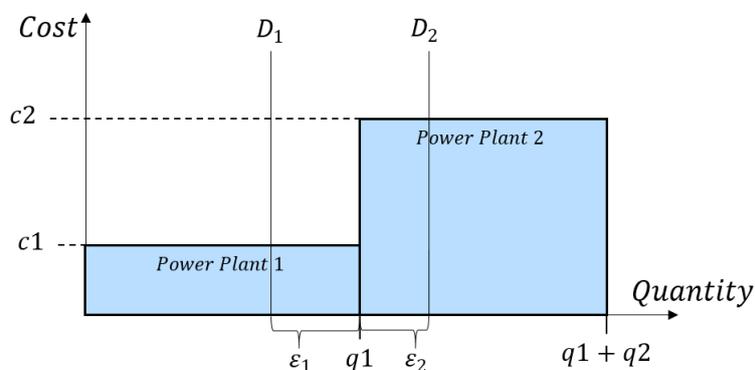


Figure 2: Example power system merit order

quantities  $q_1, q_2$  and generation costs  $c_1, c_2$ , as well as two different time steps with demand  $D_1$  and  $D_2$ . Thus, in time step 1 power plant 1 has a remaining generation capacity of  $q_1 - \varepsilon_1$ , whereas in time step 2 power plant 1 is producing at full capacity and power plant 2 has to produce the remaining demand of  $\varepsilon_2$ . Based on the merit order, market prices  $\lambda$  result as  $\lambda(t = 1) = c_1$  and  $\lambda(t = 2) = c_2$ .

We now assume that a storage system having a sufficiently large charging and discharging capacity of  $P_{max}^{ST}$  as well as a storage capacity of  $W_{max}^{ST}$  is connected to this power system. Without loss of generality, we assume that the storage is lossless and  $P_{max}^{ST} = W_{max}^{ST} > D_2 - D_1 = \varepsilon_1 + \varepsilon_2$ . Obviously, the welfare-optimal, cost minimal storage operation comprises a charging of the storage in time step 1 and discharge in time step 2 with a quantity equal to  $\varepsilon_2$ . In that case, only the cheaper power plant 1 generates power and the market price in both time steps is equal to  $c_1$ .

However, in a LR of the problem, the storage only optimizes itself given the corresponding multipliers  $\lambda$  representing the market prices. The basic feasible solutions of linear optimization problems are extreme points of the feasible region. Therefore, the storage subproblem in our example will always charge the maximum amount of power  $P_{max}^{ST} > \varepsilon_1 + \varepsilon_2$  in time step 1 and discharge the same amount in the next time step given  $\lambda(t = 1) < \lambda(t = 2)$ . Given these subproblem solutions, the LR of the EULR problem, where we relax the market coupling constraint, will never result in the optimal amount  $\varepsilon_2$ . This is due to the cut of the optimal polyhedra of subproblems with the coupling constraints in the main problem that is not considered in the relaxed subproblem, thus changing the extreme points. One can easily see how this leads to

divergent behavior of the LR methodology (due to the change in storage schedules if  $\lambda(t = 1) < \lambda(t = 2)$  in one iteration of the LR and  $\lambda(t = 1) > \lambda(t = 2)$  in the next, see Table 1).

	$\lambda(t = 1) < \lambda(t = 2)$	$\lambda(t = 1) > \lambda(t = 2)$
$P^{ST}(t = 1)$	$-P_{max}^{ST}$	0
$P^{ST}(t = 2)$	$P_{max}^{ST}$	0

Table 1: Example storage schedules (with no initial charge) for price-based storage optimization

While this divergence can be addressed by adjustments to the multiplier update methods, with these methods, producing the optimal primal solution is not achievable with the LR, but a post-processing step is required.

This structural behavior in LR has no significant impact in energy system models dominated by conventional power plants and hydro storage systems (HSS) because for HSS the storage capacity is significantly lower than the pumping and turbine capacities ( $P_{max}^{HSS} \ll W_{max}^{HSS}$ ). Therefore, HSS show a more seasonal behavior and are less sensitive to price changes based on the adjustment of Lagrangian multipliers.

In contrast, time-shifting flexibility from DERs such as battery storage systems, electric vehicles, or heating pumps in combination with heat storage systems usually is more sensitive to price changes. The installed power of such DERs compared to their storage capacity is higher than for HSS, meaning they can fully (dis-)charge in a few hours. They are, therefore, used for short-term schedule optimization within a few hours a day, leading to a more price-sensitive behavior. This problem is aggravated by a higher number of DERs that all respond to the same price signal of Lagrangian multipliers with high concurrency. Thus, the high technical flexibility and price sensitivity leads to a volatile behavior of DERs between iterations depending on the adjustment of these Lagrangian multipliers.

To address this problem, we extend the EULR market coupling subproblem, which is used for the subgradient determination, by a column generation approach similar to the process in Dantzig-Wolfe decomposition [10, 11]. DER optimization problems can be approximated by linear programs with high accuracy, leading to convex DER optimization problems. The idea behind our extension is based on the property that any convex combination of iteration solutions form a new feasible solution because the corresponding subproblem is convex.<sup>1</sup> Each column represents a primal solution of the DER subproblem optimization in each Lagrangian iteration. The combination of iteration results as the “iteration bid” of the DERs within the market coupling problem can be seen as an approximation of the optimal dispatch of those DERs in the original problem. We can describe the idea of using convex combinations of these columns to generate new solutions with the use of previously mentioned example in Table 1. In the example, the iteration result vectors are described for the first iteration  $P_{l=1}^{ST} = [-P_{max}^{ST}, P_{max}^{ST}]$ , the second iteration  $P_{l=2}^{ST} = [0, 0]$  and the welfare-optimal schedule is equal to  $P^{ST,*} = [-\varepsilon_2, \varepsilon_2]$ . Then, we can generate a solution from the convex combination of the iteration schedules  $P^{ST,*} = \varepsilon_2/P_{max}^{ST} \cdot P_{l=1}^{ST} + (1 - \varepsilon_2/P_{max}^{ST}) \cdot P_{l=2}^{ST}$ , which gives the welfare-optimal solution  $P^{ST,*}$  in this particular example.

The rest of this report is structured as follows: First, the the original EULR market coupling subproblem is presented. Next, a description of our algorithmic extension of

<sup>1</sup>A convex combination is a linear combination of vectors in a convex space with the additional constraint that the linear factors are positive and the sum of factors is equal to one.

the EULR and its relation to Dantzig-Wolfe decomposition is given. Subsequently, the extended version of the EULR market coupling subproblem is formulated. Finally, a case study for our new method as well as a discussion is presented.

## 2 EULR Market Coupling Formulation

In this section, we present the EULR market coupling subproblem formulation based on [3]. The market coupling subproblem is an approximation of EUPHEMIA market coupling algorithm [12]. The generic formulation of the original unit commitment problem is presented by (1). It can be described with a set of variables for the power plants, HSS, and renewables  $\mathbf{x}$  and a set of variables  $\mathbf{y}$  determining the optimal electricity exchange between market areas. The market coupling constraints (with Lagrangian multiplier  $\lambda$ ) are relaxed within the EULR approach, allowing for a decomposition of the model.

$$\begin{aligned} \min c^T \mathbf{x} \\ \text{s.t.} \\ \mathbf{A}\mathbf{x} \leq b \\ \mathbf{B}\mathbf{x} + \mathbf{D}\mathbf{y} \leq d(\lambda) \end{aligned} \quad (1)$$

Within the EULR approach, in each iteration of the LR, the market schedules (as a subset of  $\mathbf{x}$ ) of the different subproblems (representing power plants, HSS, renewables, etc.) are taken as inputs for the market coupling. In contrast to classical LR, the subproblem concerning the market coupling variables is solved after the other subproblems (cf. Fig. 1). The iteration schedules  $\mathbf{x}$  are interpreted as market bids  $o$  of the different plants that can also be accepted partially within the market coupling, thus allowing for an optimized determination of subgradients of the relaxed constraints. Furthermore, non-variable components such as electricity demand are also taken into account as bids. The different schedules and demands represent quantity bids  $q_o$  (with  $q_o > 0$  for generation and  $q_o < 0$  for load). The price component  $p_o$  of the bids  $o \in O$  is parameterized based on either marginal costs, the last iteration's price  $\lambda$ , regulatory price components or cost of lost load depending on the type of bidding technology [3]. Each bid  $o$  in every time step  $t \in T$  is assigned a corresponding market area  $m \in M$  based on the localization of the bidding asset and a continuous decision variable  $\mathbf{A}_o$  defines the degree of acceptance of each bid. As in [12], the objective of the market coupling comprises a welfare-maximization of the accepted bids and it is given by the equation (2). Furthermore, the exchange  $\mathbf{F}_{i,j}$  between two market areas  $i$  and  $j$  is only restricted by net transfer capacities (NTC) between both market areas  $F_{i,j}^{max}$  (5). For simplicity, only the NTC approach for market coupling is shown in this section with equations (4) - (5). Flow-based market coupling can also be considered within this approach with adjusted market-coupling constraints [13]. The balance of exchange between all market areas is equal to zero as given in equation (6).

$$\max \sum_{t \in T} \sum_{o \in O(t)} -(q_o \cdot p_o) \cdot \mathbf{A}_o \quad (2)$$

s.t.

$$\sum_{o \in O(m,t)} q_o \cdot \mathbf{A}_o - \mathbf{nex}_m(t) = 0, \forall m \in M, t \in T \quad (3)$$

$$\mathbf{nex}_m(t) = \sum_{j \in M \setminus m} \mathbf{F}_{m,j}(t) - \mathbf{F}_{j,m}(t), \forall m \in M, t \in T \quad (4)$$

$$0 \leq \mathbf{F}_{i,j}(t) \leq F_{i,j}^{max}, \forall t \in T, \{i \in M, j \in M | i \neq j\} \quad (5)$$

$$\sum_{m \in M} n e x_m(t) = 0, \forall t \in T \quad (6)$$

$$0 \leq \mathbf{A}_o \leq 1, \forall o \in O \quad (7)$$

Note that in this simplified market coupling approach, no time-coupling of bids is considered. Within the EULR approach, the market coupling problem (2)-(7) is used to determine the LR subgradients by evaluating the not accepted supply and demand (oversupply and overdemand). Thus, the adjustment of LR multipliers is based on the result of the EULR market coupling problem.

To integrate DER submodels and to improve the convergence of the process, the EULR market coupling subproblem is expanded by approaches taken from Dantzig-Wolfe decomposition.

### 3 Algorithmic extension of EULR approach for DERs

The general idea for integrating the DER iteration results into the EULR market coupling is to approximate the primal DER solution of the un-relaxed problem. To that end, we extend the original EULR approach and the original market coupling formulation by integrating DER iteration results into the EULR market coupling.

Firstly, the DER models are integrated into the EULR model as subproblems of the LR [9]. The DER optimization problems are solved at each iteration given the electricity market price (LR multiplier) of that iteration, analogously to power plant subproblems.

The core component of the DER integration approach is the approximation of the optimal DER solution through a convex combination of DER iteration solutions.

This component is inspired from the column generation methodology applied within the Dantzig-Wolfe decomposition framework and the volume algorithm. In [14], the volume algorithm is proposed to generate optimal primal solutions within a subgradient method. By calculating an exponentially weighted moving average  $s_l$  of the subproblem solutions  $x_l$  over the LR iterations  $l$ , the optimal primal solution of the problem is approximated with help of the weighting factor  $\alpha$  as in the equation (8).

$$s_l = (1 - \alpha) \cdot s_{l-1} + \alpha \cdot x_l, \forall l \in L \quad (8)$$

This approach is described in [14] “as a fast way to approximate Dantzig-Wolfe decomposition.”

In Dantzig-Wolfe decomposition, the original problem is reformulated such that the complicating constraints (that are relaxed in LR) are part of the master problem. However, a restricted master problem that includes a restricted set of variables is solved iteratively instead of solving the master problem with a large number of variables at once. At each iteration, the restricted master problem dual solution is used to generate the additional columns for the restricted master problem in the next iteration by solving the pricing problem.<sup>2</sup>

We take the idea of using the convex combination of subproblem iteration solutions and apply it to the LR iteration solution of the DER subproblems within the market coupling. Note that we apply the usage of convex combinations only to DER subproblems, especially since the power plant formulations are non-convex (mixed integer).

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<sup>2</sup>If the relaxed constraints of LR correspond to the constraints of the Dantzig-Wolfe master problem, the optimal dual variables of the Dantzig-Wolfe master problem correspond to the optimal Lagrangian multipliers of the LR and the subproblems of both decomposition algorithms correspond to each other [15].

Therefore, our approach is a heuristic replicating the ideas of column generation method used within the Dantzig-Wolfe decomposition for the DER iteration solutions.

The approach to offer convex combinations within the market coupling can be interpreted as a new kind of market bid of DERs where some flexibility is given to the market coupling algorithm.

The formulation of the extended market coupling problem is presented in the next chapter.

## 4 Time-Coupled EULR Market Coupling Formulation with consideration of DER bids

This section presents the extended market coupling formulation incorporating the convex combination of the different DER iteration results to counteract the diverging behavior of DER subproblems. As noted earlier, the convex combination of iteration solutions forms a new feasible solution if the corresponding subproblem is convex, which is the case for our DER subproblems. Furthermore, a convex combination of only a subset of the subproblem variables remains valid since the rest of the variables can be analyzed and calculated ex-post. It is not necessary to include all variable results for all iterations to form a feasible convex combination which further leads to reduced information requirements. Therefore, only the market quantities of each DER  $d$  in each time-step  $t$  have to be considered but other variables can be neglected. Given a DER solution  $q_l^d$  of a given iteration  $l$  defined in equation (9).

$$q_l^d = [q_l^d(1), \dots, q_l^d(T)], \forall l \in L, d \in D \quad (9)$$

It is obvious that to ensure the technical feasibility the whole solution vector  $q_l^d$  has to be considered simultaneously.<sup>3</sup> Thus, the time-decoupled market coupling formulation given by equations (2)-(7) has to be reformulated and expanded to take into account the time-coupling of the DER iteration solutions.

For this purpose, a new variable for acceptance ratio notated as  $\mathbf{A}_l^d$  is introduced for each DER iteration solution. Note that this acceptance ratio is not defined for one time step but for the whole iteration solution vector (9) such that the acceptance of the iteration solution is equal in all time steps. Then, the sum of acceptance over all DER iteration solutions has to be equal to one to form a convex combination:

$$\sum_{l \in L} \mathbf{A}_l^d = 1, \forall d \in D \quad (10)$$

$$0 \leq \mathbf{A}_l^d \leq 1, \forall d \in D, l \in L \quad (11)$$

Thus, the term  $q^d = \sum_{l \in L} q_l^d \cdot \mathbf{A}_l^d$  forms the final DER result. To account for a non-acceptance of DER bids, two additional "slack bids" for generation  $q_m^{SG}(t)$  and load  $q_m^{SL}(t)$  are introduced per market area  $m$  with DERs and per time step  $t$ . Then a residual (accepted) demand  $res_m(t)$  from DERs can be calculated as follows:

$$res_m(t) = q_m^{SG}(t) \cdot \mathbf{A}_m^{SG}(t) + q_m^{SL}(t) \cdot \mathbf{A}_m^{SL}(t) + \sum_{d \in D(m)} \sum_{l \in L} q_l^d(t) \cdot \mathbf{A}_l^d, \forall m, t \quad (12)$$

The non-supplied bids of DERs is then measured by the slack generation  $q_m^{SG}(t) \cdot \mathbf{A}_m^{SG}(t)$  and slack load  $q_m^{SL}(t) \cdot \mathbf{A}_m^{SL}(t)$ .

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<sup>3</sup>In theory, this also holds for power plants and HSS. But, due to their lower sensitivity to price changes, their bids are more consistent and the resulting acceptance ratio is higher (closer to one).

By expanding the equation (3) with  $\mathbf{res}_m(t)$ , we can integrate the time-coupled DER bids into the market coupling formulation:

$$\sum_{o \in \mathcal{O}(m,t)} q_o \cdot \mathbf{A}_o + \mathbf{res}_m(t) - \mathbf{nex}_m(t) = 0, \forall m, t \quad (13)$$

To correctly account for the overdemand and oversupply in each market zone as subgradients of the Lagrange multiplier adjustment process (cf. Figure 1), the slack generation and load are placed last in the merit order by parameterization of their corresponding price bids (e.g.  $p^{SG} > p_o, \forall o$ ). Furthermore, the iteration price bids of DERs are defined by the current market price  $\lambda_m(t)$ <sup>4</sup>:

$$p_l^d(t) = -\lambda_{m,l}(t), \forall l \in L, t \in T, m \in M, d \in D(m) \quad (14)$$

The final time-coupled market coupling formulation is then given by (15)-(17):

$$\begin{aligned} \max \sum_t \sum_{o \in \mathcal{O}(t)} -(q_o \cdot p_o) \cdot \mathbf{A}_o \\ + \sum_t \sum_m -(q_m^{SG}(t) \cdot p^{SG}) \cdot \mathbf{A}_m^{SG}(t) - (q_m^{SL}(t) \cdot p^{SL}) \cdot \mathbf{A}_m^{SL}(t) \\ + \sum_t \sum_m \sum_{d \in D(m)} \sum_{l \in L} -(q_l^d(t) \cdot p_l^d(t)) \cdot \mathbf{A}_l^d \end{aligned} \quad (15)$$

s.t.

$$Eq.(4) - Eq.(7) \quad (16)$$

$$Eq.(10) - Eq.(13) \quad (17)$$

The number of variables can be reduced if all DERs in one market area first form one combined iteration bid (by summarizing the individual iteration bids). The combined coupling result can then later be disaggregated to the individual DER results. This simplification has the further advantage that all DERs in one market area are treated equally within the market coupling algorithm.

To ensure complete technical feasibility for the convex combination of DER bids, the market coupling period and the subproblem optimization period have to be identical. However, a calculation of the market coupling problem in independent time slices reduces the computational complexity of the market coupling problem but increases the risk of technical infeasibilities of the combined DER coupling vector.

## 5 Case Study

The proposed method of this report is demonstrated with a test case based on the data from plan4res case study 1 [16]. One of the aims for the he project plan4res is to develop an end-to-end planning and operation tool, composed of a set of optimization models based on an integrated modelling of the pan-European Energy System [17]. Hence, a modeling framework comprising of interrelated models have been developed by the project [18]. There are three case studies in the project that highlight tool's adequacy and relevance [19]. Besides, a public data set has been constructed from the data used by these case studies [20]. For our test case, we only consider DERs in Germany to reduce computational requirements. Furthermore, for our case study only electric vehicles (with no consideration for vehicle-to-grid) and power-to-heat systems in combination with heat storage systems are considered as DERs. The electric

<sup>4</sup>In contrast to Dantzig-Wolfe decomposition, where the objective value for  $\mathbf{A}_l^d$  is defined by the objective value of the subproblems' iteration solution. The parameterization here is to ensure consistency with the rest of the market coupling model.

vehicles have a capacity of up to 355 Gigawatt (GW) and the capacity of the power-to-heat system amounts to 20 GW. Based on the bottom-up data from [16], the DER technologies are aggregated on the German high-voltage substation level and optimized as local virtual power plants (LVPP) leading to 4324 DER or LVPP subproblems.

For our case study, we now compare the following three approaches for integrating DERs into the EULR model.

- C1 EULR Market Coupling formulation solution with basic LR (“basic LR”): We solve the EULR market coupling model with the LR where only the iteration’s DER result is considered within the market coupling as described in equations (2)-(7),
- C2 EULR Market Coupling formulation solution with the volume algorithm (“Volume algorithm”): We use the volume algorithm with an  $\alpha$ -Value of 20% for approximating primal optimal solutions of the DER subproblems as presented in Section 3 as input of the market coupling (2)-(7).
- C3 Time-Coupled Market Coupling (TCMC): We solve the TCMC formulation presented in Section 4 with an additional minimum acceptance of  $A_{|L|}^d \geq \alpha = 20\%$  for the current iteration solution to increase comparability with the volume algorithm.

For our case study, we compare the results of the LR after 100 iterations. Empirically, the convergence improvements after 100 iterations are only minor in contrast to the additional computational effort of another LR iteration.

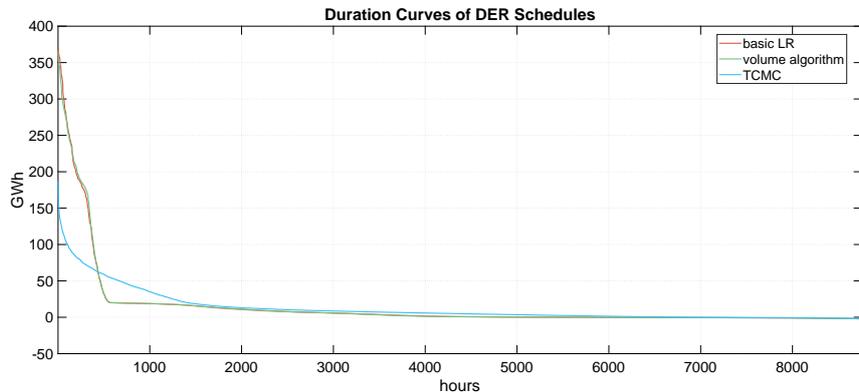


Figure 3: Duration curves of DER schedules

First, we compare the resulting primal solutions of the DERs, i.e., DER load within the market coupling. In Figure 3, the duration curves of the bidding schedules for the volume algorithm and for the basic LR are shown as well as the convex combination  $\sum_{l \in L} q_l^d \cdot A_l^d$  resulting from the TCMC approach in Section 4. It can be seen that the schedules from C3 lead to fewer peaks in demand and a smoother demand profile. By forming combinations of different iteration bids with extreme peaks, the DER schedules are better adjusted to the general system properties, e.g. the residual load of the system (defined as inflexible demand minus volatile renewable generation).

Furthermore, C3 leads to less volatility of DER bids between the different iterations. This is shown in Figure 4 where the duration curves of the differences between iteration bids (i.e. a sorted order of the vectors  $\{\Delta q_l^d | \Delta q_l^d = q_l^d - q_{l-1}^d, l = 2, \dots, |L|\}$ )

are plotted for the different iterations and the three approaches. It can be seen that the maximum absolute deviations of the schedules between the iterations are significantly lower for C3 and highest for C1. The deviation of schedules between iterations can be seen as an indication of convergence problems, since these deviations are reactions to changes of the Lagrangian multipliers. Because DERs are more sensitive to the changes of Lagrangian multipliers than the rest of the system, i.e. power plants, HSS, renewable plants, the DER deviations increase the Lagrangian subgradient (oversupply and overdemand per market area) instead of decreasing it.

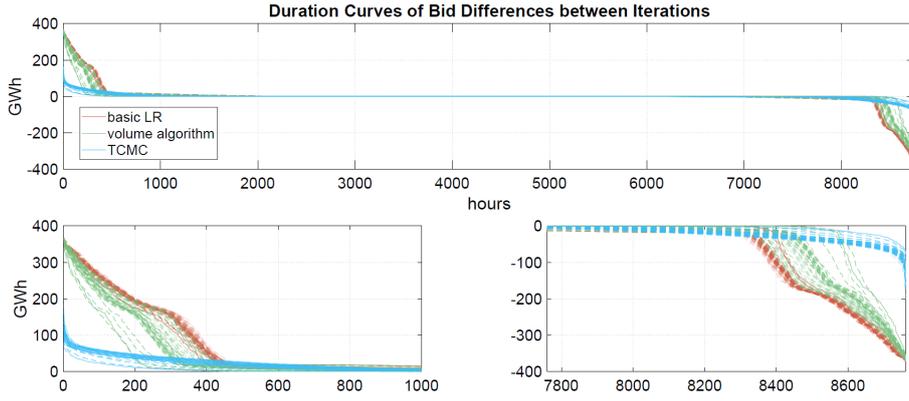


Figure 4: Duration curves of differences between Lagrangian iterations

As a measure of convergence, we compare the final cost for the three approaches. Non-supplied load demands (both from the static load and DER load bids) are considered within this cost comparison with a cost-factor of  $400\text{EUR}/\text{MWh}$ . The generation costs comprise the fuel and start-up costs of conventional power plants. The resulting costs for the three approaches are plotted in Figure 5. C1 results in the lowest generation cost in the expense of a large amount of unsupplied demand, as reflected in the high cost for loss-of-load. C3 leads to a smaller generation cost in comparison to C2 and less loss-of-load, leading to the smallest total generation costs of all three approaches. In conclusion, C3 gives the best solutions compared to C1 and C2 both in terms of cost and solution quality.

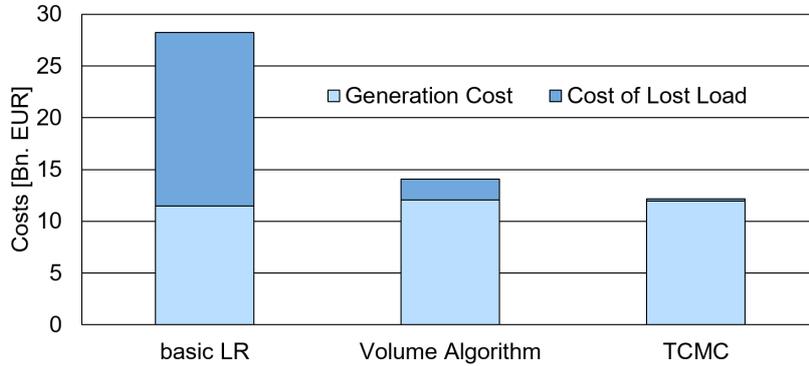


Figure 5: Comparison of Generation and Loss-of-Load costs in the German market area

## 6 Conclusion

LRs of pan-European unit commitment models are an established approach for modeling the pan-European day-ahead electricity market in energy planning and analysis processes. However, the integration of models for distributed, small-scale technologies such as electric vehicles into LR models might lead to convergence problems due to the concurrency of the behavior of the corresponding technology submodels. Therefore, in this report, the existing EULR model [3] is extended to improve the solution convergence when incorporating DER optimization models as subproblems. The results of an exemplary case study show the effectiveness of this approach.

The approach and results presented in this report allow for further investigations and improvements. The interpretation of the EULR model as a welfare-optimization with consideration of profit optimization of individual actors assumes the optimal subproblem dispatch for given market prices (Lagrangian multipliers). Therefore, any deviation of the convex combination of DER bids  $q^d$  from the last iteration bid  $q_{|L|}^d$  represents a non-optimal market schedule. To align the DER actor perspective with the market coupling approach, two measures are proposed: Firstly, a minimum acceptance of the last iteration schedule as carried out in the case study. Secondly a filter of iteration bids considered within the market coupling such that the revenue is at most  $b\%$  smaller than the current iteration revenue:  $\sum_t q_t^d(t) \cdot \lambda_t \geq (1 - b)\% \cdot \sum_t q_{|L|}^d(t) \cdot \lambda_t$ . This ensures that the possible resulting bid combination  $q^d$  is not too disadvantageous from a DER operator perspective. Additionally, the time-coupling of DER schedules within the market coupling EULR subproblem is the first step for more detailed modeling of the EUPHEMIA algorithm [12] within the EULR model and can further be extended. Furthermore, the TCMC approach using methods from Dantzig-Wolfe decomposition and LR presented in this report is only a heuristic approach to improve the convergence process which can further be improved with a more detailed mathematical analysis.

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## Abbreviations

DER	Distributed Energy Resource
EULR	EUropean Lagrangian Relaxation
EUR	Euro
GW	Gigawatt
HSS	Hydro Storage System
LVPP	Local Virtual Power Plant
LR	Lagrangian Relaxation
NTC	Net Transfer Capacity
MWh	Megawatthour
TCMC	Time-Coupled Market Coupling

# Nomenclature

## Indices and Sets:

$m \in M$	index and set of market areas
$o \in O$	index and set of market bids
$t \in T$	index and set of time period
$d \in D$	index and set of DER subproblems
$l \in L$	index and set of LR iterations

## Parameters and Constants:

$A, B, C$	Generic constraint matrices
$b, d$	Generic constraint bounds
$q_o$	quantity component of bid $o$
$p_o$	price component of bid $o$
$F_{i,j}^{max}$	maximum exchange (NTC) between market areas $i$ and $j$
$\alpha$	weighting factor for the volume algorithm
$\lambda$	Lagrangian multipliers/ electricity prices

## Variables:

$A_o$	Acceptance ratio for bid $o$
$nex_m$	Netto export of market area $m$
$F_{m,j}$	Export from market area $m$ to market area $j$
$F_{j,m}$	Import from market area $j$ to market area $m$
$A_l^d$	Acceptance ratio for iteration solution $l$ of DER $d$
$res_m$	Residual accepted bid of DERs in market area $m$
$x$	Generic variables of power plants
$y$	Generic market coupling variables

# References

- [1] M. Ventosa, A. Baillo, A. Ramos, and M. Rivier (2005). Electricity market modeling trends. *Energy policy*, 33(7), 897-913.
- [2] M. Nobis, L. Wyrwoll, A. Moser and S. Raths, "Impact of market-coupling on electricity price modeling in fundamental unit-commitment approaches", 2020 6th IEEE International Energy Conference (ENERGYCon), Gammarth, Tunis, Tunisia, 2020, pp. 740-743, doi: 10.1109/ENERGYCon48941.2020.9236434.
- [3] S. Raths, "Ein Marktsimulationsverfahren für einen dezentral geprägten Strommarkt", PhD Thesis, RWTH Aachen University, 2020.
- [4] J.F. Benders. "Partitioning procedures for solving mixed-variables programming problems". *Numerische Mathematik*, 4, 1962, pp.238-252.
- [5] R. Rahmaniani, T.G. Crainic, M. Gendreau, and W. Rei, "The Benders decomposition algorithm: A literature review", *European Journal of Operational Research*, Volume 259, Issue 3, 2017, pp.801-817.
- [6] G. A. Dourpbois and P. N. Biskas, "European Power Exchange day-ahead market clearing with Benders Decomposition", 2014 Power Systems Computation Conference, Wroclaw, Poland, 2014, pp. 1-7, doi: 10.1109/PSCC.2014.7038452
- [7] C. Barnhart, E. L. Johnson, G. L. Nemhauser, M. W. P. Savelsbergh, and P. H. Vance. "Branch-and-price: Column generation for solving huge integer programs". *Operations Research*, 46:316-329, 1996.
- [8] T. Drees, "Simulation des europäischen Binnenmarktes für Strom und Regelleistung bei hohem Anteil erneuerbarer Energien", PhD Thesis, RWTH Aachen University, 2016.

- [9] M. Nobis and T. Kulms. “Evaluating regulatory measures in the German energy transition–A European multimodal market optimization approach including distributed flexibilities”. Local Energy, Global Markets, 42nd IAEE International Conference, May 29-June 1, 2019. International Association for Energy Economics, 2019.
- [10] G. B. Dantzig and P. Wolfe. “Decomposition principle for linear programs”. Operations Research, 8(1):101–111, 1960.
- [11] G. B. Dantzig and P. Wolfe. “ The decomposition algorithm for linear programs”. Econometrica, 29(4):767–778, 1961.
- [12] NEMO Committee. EUPHEMIA Public description-Single Price Coupling Algorithm. Tech. Rep., April, 2019.
- [13] L. Wyrwoll, K. Kollenda, C. Müller and A. Schnettler. “Impact of flow-based market coupling parameters on European electricity markets”. In 2018 53rd International Universities Power Engineering Conference (UPEC) (pp. 1-6). IEEE, 2018
- [14] F. Barahona and A. Ranga. “The volume algorithm: producing primal solutions with a subgradient method”. Mathematical Programming 87.3 (2000): 385-399.
- [15] D. Huisman et al. “Combining column generation and Lagrangian relaxation”. Column generation. Springer, Boston, MA, 2005. 247-270.
- [16] plan4res, “Deliverable D4.2 - Dataset for case study 1 and document 'how this dataset was built'”, 2020, plan4res.eu
- [17] plan4res Project Website. Available online: <https://www.plan4res.eu> (accessed on 19.03.2020).
- [18] D. Beulertz, S. Charousset-Brignol, S. Most, S. Giannelos, I. Yueksel-Erguen. “Development of a Modular Framework for Future Energy System Analysis”. 54th International Universities Power Engineering Conference (UPEC), 2019, pp. 1-6, doi: 10.1109/UPEC.2019.8893472.
- [19] D. Most, S. Giannelos, I. Yueksel-Erguen, D. Beulertz, U. Haus, S. Charousset-Brignol, A. Frangioni. “A Novel Modular Optimization Framework for Modelling Investment and Operation of Energy Systems at European Level”. *ZIB-Report 20-08* **2020**.
- [20] Overview-Scope of Case Study 1. Available online: <https://www.plan4res.eu/wp-content/uploads/2019/06/plan4res-Definition-Case-Studies-Summary-CS1.pdf> (accessed on 20.04.2021)
- [21] plan4res public data repository. Available online:<https://zenodo.org/communities/plan4res/?page=1&size=20> (accessed on 20.04.2021)